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Information Fusion Based on Information Entropy in Fuzzy Multi-source Incomplete Information System

Weihua Xu[1] · Mengmeng Li[1] · Xizhao Wang[2]

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Abstract With the development of society, although the way that people get information more and more convenient, the information which people get may be incomplete and has a little degree of uncertainty and fuzziness. In real life, the incomplete fuzzy phenomenon of information source exists widely. It is extremely meaningful to fuse multiple fuzzy incomplete information sources effectively. In this study, a new method is presented for information fusion based on information entropy in fuzzy incomplete information system and the effectiveness of the new method is verified by comparing the average fusion method. Then, an illustrative example is delivered to illustrate the effectiveness of the proposed fusion method. Finally, we have also tested the veracity and validity of this method by experiment on a dataset from UCI. The results of this study will be useful for pooling the uncertain data from different information sources and significant for establishing a distinct direction of the fusion method.

Keywords Fuzzy set theory · Information entropy · Incomplete information system · Multi-source information fusion

✉ Weihua Xu
chxuwh@gmail.com

1 School of Mathematics and Statistics, Chongqing University of Technology, Chongqing 400054, People's Republic of China

2 College of Computer Science & Software Engineering, Shenzhen University, Shenzhen, Guangdong 518060, People's Republic of China

1 Introduction

With the continuous progress of sciences and technology, people's lives are continually improving, and today's society has become the era of data explosion. Everyday, people can accept a large number of data and information from various aspects, but not every kind of data or information is clear, accurate, and complete. We have an obligation to filter the data and information, which we need from the vast amount of data and information. As we get into the information age, information acquisition, information comprehensive analysis and processing, and information fusion have become the research focus in the field of information technology.

Everyday language and decision-making are not generally deterministic but are usually characterized by some level of fuzziness or uncertainty. In 1965, fuzzy set theory [1], originated by Zadeh, is a mathematical tool to deal with uncertainty in an information system. Fuzzy set theory, which was triggered by these considerations, provides a conceptual framework for solving knowledge representation and classification problems in an ambiguous environment. The theory of fuzzy set is an extension of classical crisp set theory for the study of intelligent systems characterized by fuzzy information. Recently, a lot of topics have been widely studied using rough set and fuzzy set. Dubois and Prade [2] proposed the fuzzy rough set for dealing with fuzzy information system. It has been shown to be useful in the fields of data mining, pattern recognition, knowledge discovery, and so on.

Information system is the carrier of our most access to information resources. It will inevitably become the principal object of information science. Information systems with huge amount of information have significant uncertainties. These uncertainty measures are the important issues of data mining and knowledge discovery. Wang et al. [3, 4] discussed the uncertainty in learning from big data and investigated also the uncertainty in learning from Big Data-Editorial. In real life, all the information which we can get is all not clear and complete. Most of the information systems we can get are fuzzy and incomplete. In recent years, with the need for practical engineering applications, it has grown up to become a hot research topic to obtain knowledge from incomplete information system [5–13] using rough set theory [14–17]. Fu et al. [18] researched attribute reduction algorithm-based information entropy in incomplete information systems. Ashfaq et al. [19] investigated fuzziness-based semi-supervised learning approach for intrusion detection system. Wang et al. [20] discussed that fuzziness-based sample categorization for classifier performance improvement. Yang et al. [21] studied rough set model based on variable parameter classification in incomplete information systems. Wang et al. [22, 23] investigated the relationship between generalization abilities and fuzziness of base classifiers in ensemble learning and discussed particle swarm optimization for determining fuzzy measures from data. He et al. [24] studied the fuzzy nonlinear regression analysis using a random weight network.

Information fusion technology is based on multisensor data fusion, and related information from the associated database, which can obtain more information with higher accuracy than a single sensor. Multisensor information fusion technology increases the reliability of the measurements and, to improve the reliability of the system, has been widely used in industries, military, and so on; people in different fields present different sensor information fusion methods. Ribeiro et al. [25] proposed an algorithm of information fusion. Lin et al. [26] studied optimistic and pessimistic multi-granulation fusion functions based on multi-granulation rough set theory. Zhou et al. [27] investigated a systematic method established on the basis of evidence theory in multi-source information system for variation source identification of deep hole boring process. Cai et al. [28] researched the multi-source information fusion-based fault diagnosis of ground-source heat pump. Ma [29] provided formation drill ability prediction based on multi-source information fusion. Belur [30] investigated the architecture, algorithms, and applications of multi-source information fusion. In the information fusion system, the information provided by a single sensor may be incomplete and inaccurate and have a little degree of uncertainty and fuzziness, and some are even contradictory. In this paper, we will consider information fusion based on information entropy in fuzzy multi-source incomplete information system. And it is concluded that the information entropy fusion method is more effective when compared to the mean value fusion method.

The rest of this paper is structured as follows. Section 2 provides relevant basic concepts of rough set, fuzzy set, and conditional entropy. In Sect. 3, the conditional entropy in a multi-source fuzzy incomplete information decision system is defined, and a fusion method is proposed based on information entropy and an algorithm is designed to integrate multiple sources based on information entropy. In Sect. 4, we test the veracity and validity of this method by experiment on a dataset from UCI. Finally, Sect. 5 presents the conclusions.

In order to conveniently illustrate the key idea, we give a block diagram of the proposed approach in the following (Fig. 1).

2 Preliminaries

In this section, we review some basic concepts such as rough set theory, fuzzy set theory [1, 31], incomplete information system [5, 32–34], and conditional entropy(CE) [35].

2.1 Pawlak's rough set

An information system is a quadruple $IS = (U, AT, V, f)$. U is a nonempty finite set of objects, AT is a nonempty finite set of attributes, $V = \bigcup_{a \in A} V_a$ with V_a being the domain of a, and $f : U \times AT \rightarrow V$ is an information function with $f(x, a) \in V_a$ for each $a \in AT$ and $x \in U$. A decision system (DS) is a quadruple $DS = (U, AT \cup DT, V, f)$, where AT is the condition attribute set, DT is the decision attribute set, and $AT \cap DT = \phi$.

Given an equivalence relation R on the universe U and $X \subseteq U$, the lower approximation and upper approximation of X are defined by

$$\overline{R}(X) = \{x \in U | [x]_R \cap X \neq \emptyset\} = \cup\{[x]_R | [x]_R \cap X \neq \emptyset\},$$
$$\underline{R}(X) = \{x \in U | [x]_R \subseteq X\} = \cup\{[x]_R | [x]_R \subseteq X\}.$$

Approximation accuracy proposed by Pawlak provides the percentage of possible correct decisions when classifying objects by employing the attribute set R. Let $DS = (U, AT \cap D, V, f)$ be a decision system. $U/D = \{Y_1, Y_2, \cdots, Y_{,m}\}$ be a classification of universe U, and R be an attribute set that $R \subseteq AT$. Then, the $R-lower$ and $R-upper$ approximations of U/D are defined, respectively, as

$$\underline{R}(U/D) = \underline{R}(Y_1) \cup \underline{R}(Y_2) \cup \cdots \cup \underline{R}(Y_m)$$

$$\overline{R}(U/D) = \overline{R}(Y_1) \cup \overline{R}(Y_2) \cup \cdots \cup \overline{R}(Y_m).$$

The approximation accuracy and the corresponding approximation roughness of U/D by R are defined, respectively, as

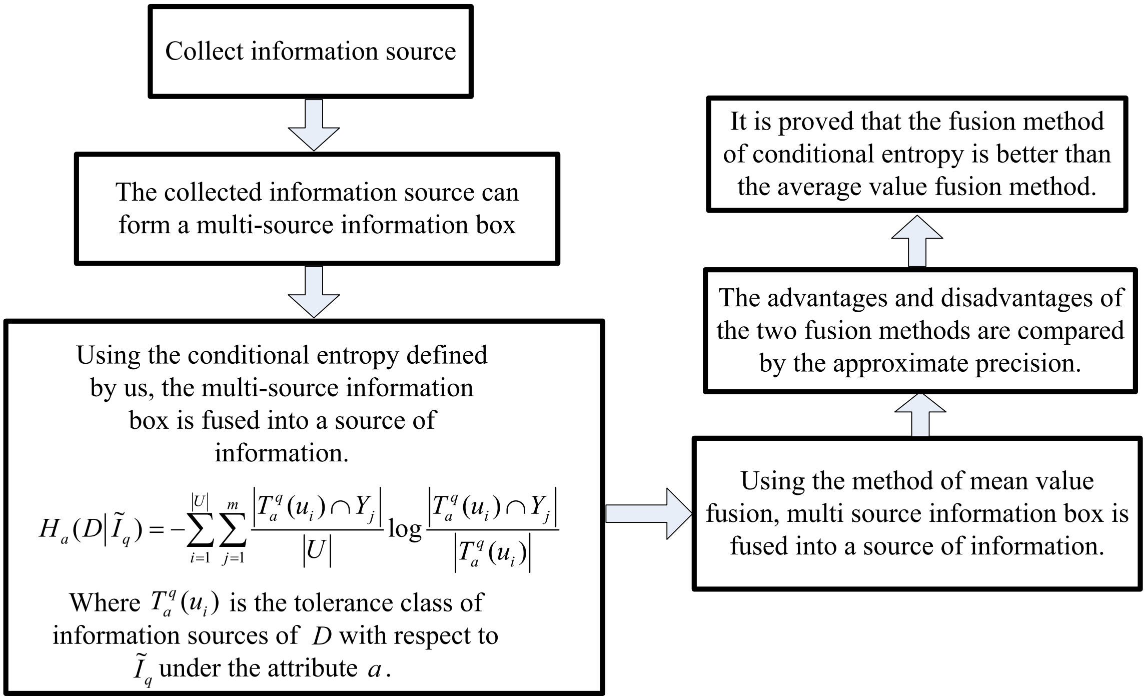

Fig. 1 Block diagram of the proposed approach

$$\alpha_R(U/D) = \frac{\sum_{Y_i \in U/D} |\underline{R}(Y_i)|}{\sum_{Y_i \in U/D} |\overline{R}(Y_i)|}$$

$$Roughness_R(U/D) = 1 - \alpha_R(U/D).$$

2.2 Fuzzy Set

Fuzzy sets are first proposed by Zadeh attaches great importance to the idea of partial membership, which departs from the dichotomy. Fuzzy set theory is a generalization of the classical set theory. Let U the so-called universe be a nonempty finite set. A fuzzy set $\tilde{X}$ of U can be expressed as

$$\tilde{X} = \{<x, \mu_{\tilde{X}}(x)> | x \in U\},$$

where $\mu_{\tilde{X}}(x) : U \rightarrow [0, 1]$, $\mu_{\tilde{X}}(x)$ is called the membership degree to $\tilde{X}$ of the object $x \in U$. Let $F(U)$ customarily denote all fuzzy sets in the universe U. Given two sets $\tilde{X}_1$, $\tilde{X}_2 \in F(U)$, for any $x \in U$, $\mu_{\tilde{X}_1}(x) \leq \mu_{\tilde{X}_2}(x)$ if and only if $\tilde{X}_1 \subseteq \tilde{X}_2$; $\mu_{\tilde{X}_1}(x) = \mu_{\tilde{X}_2}(x)$ if and only if $\tilde{X}_1 \subseteq \tilde{X}_2$ and $\tilde{X}_2 \subseteq \tilde{X}_1$.

2.3 Incomplete Information System

For an information system, if there exist $a \in AT$ and $x \in U$ such that $f(a, x)$ is equal to a missing value(denoted as "$*$"), then the information system is an incomplete information system (*IIS*). Otherwise, the information system is a complete information system (*CIS*).

Since there are missing values, the equivalence relation is not suitable for incomplete information systems. Hence, Kryszkiewicz [36, 37] defined a kind of tolerance relation for incomplete information systems. Given an incomplete information system $IIS = (U, AT, V, f)$, for any attribute subset $B \subseteq AT$, let $T(B)$ denote the binary tolerance relation between objects that are possibly indiscernible in terms of B. $T(B)$ is defined as

$$T(B) = \{(x, y) | \forall a \in B, f(a, x) = f(a, y) \ or \ f(a, x) = * \ or \ f(a, y) = *\}.$$

The tolerance class of object x with reference to an attribute set B is denoted as $T_B(x) = \{y | (x, y) \in T(B)\}$. For $X \subseteq U$, the lower and upper approximations of X with respect to B can be further defined as

$$\overline{T_B}(X) = \{x \in U | T_B(x) \cap X \neq \emptyset\},$$
$$\underline{T_B}(X) = \{x \in U | T_B(x) \subseteq X\}.$$

2.4 Conditional Entropy

In the literature [35], Dai et al. proposed a new conditional entropy to evaluate the uncertainty in incomplete decision systems. Given a decision system $DS = (U, AT \cup DT, V, f)$, $U = \{u_1, u_2, \ldots, u_n\}$. $B \subseteq AT$ is an attribute set; $U/D = \{Y_1, Y_2, \ldots, Y_m\}$. The conditional entropy of D with respect to B is defined by

$$H(D|B) = -\sum_{i=1}^{|U|}\sum_{j=1}^{m}\frac{|T_B(u_i)\cap Y_j|}{|U|}\log\frac{|T_B(u_i)\cap Y_j|}{|T_B(u_i)|}.$$

2.5 Multi-source Fuzzy Incomplete Information System

Let us take a look at the scenario when we obtain information regarding a set of objects from different sources. Information from each source is collected in the form of the above information system [17], and thus a family of single information systems with the same domain is obtained and called a multi-source information system which is formulated as follows (see [26]).

A multi-source fuzzy incomplete information system ($MFIIS$) can be defined as $MFIIS = \{FIIS_i | IIS_i = (U, \widetilde{AT_i}, \{(V_a)_{a\in\widetilde{AT_i}}\}, f_i)\}$, where

(1) $FIIS_i$ is a fuzzy incomplete information system of each subsystem ;
(2) U is a finite nonempty set of objects;
(3) $\widetilde{AT_i}$ is a finite nonempty fuzzy set of attributes of each subsystem;
(4) $\{V_a\}$ is the value of the attribute $a \in \widetilde{AT_i}$;
(5) $f_i : U \times \widetilde{AT_i} \to \{(V_a)_{a\in\widetilde{AT_i}}\}$ such that for all $x \in U$ and $a \in \widetilde{AT_i}$, $f_i(x,a) \in V_a$.

3 Information Fusion Based on Information Entropy in Fuzzy Multi-source Incomplete Information System

Nowadays, a great deal of data are openly accessible through Internet, and the volume of information is constantly increasing. In the process of information explosion, many fuzzy and incomplete information sources are generated. It is imperative how to make full use of the fuzzy and incomplete information from multiple sources. So multi-source fuzzy incomplete information fusion is a desirable research direction. The purpose of information fusion is to obtain more comprehensive information.

For each table of information box that is fuzzy incomplete, we propose a novel fusion method.

Definition 3.1 Let I be a fuzzy incomplete information system (MFIIS) and $U = \{x_1, x_2, ..., x_n\}.\forall a \in \widetilde{AT}, x_i, x_j \in U$, and we define the distance between any two objects in U under the attribute a as follows:

$$dis_a(x_i,x_j) = \begin{cases} 0, & if f(x_i,a) = * \quad or \quad f(x_j,a) = * \\ |f(x_i,a) - f(x_j,a)| & else. \end{cases}$$

Definition 3.2 Given a fuzzy incomplete information system $MFIIS = (U, \widetilde{AT}, V, f)$, for any attribute $a \in \widetilde{AT}$, let T(a) denote the binary tolerance relation between objects that are possibly indiscernible in terms of a. T(a) is defined as

$$T(a) = \{(x,y) | dis_a(x,y) \leqslant L_a\},$$

where L_a indicates the threshold associated with attribute a. The tolerance class of object x with reference to an attribute a is denoted as $T_a(x) = \{y | (x,y) \in T(a)\}$.

Definition 3.3 Given a fuzzy incomplete information system $MFIIS = (U, \widetilde{AT}, V, f)$, for any attribute subset $\widetilde{B} \subseteq \widetilde{AT}$, let $T(\widetilde{B})$denote the binary tolerance relation between objects that are possibly indiscernible in terms of $\widetilde{B}$. *$T(\widetilde{B})$ is defined as*

$$T(\widetilde{B}) = \bigcap_{a\in\widetilde{B}} T(a).$$

The tolerance class of object x *with reference to an attribute set* $\widetilde{B}$is denoted as $T_{\widetilde{B}}(x) = \{y | (x,y) \in T(\widetilde{B})\}$.

In the literature [35], Dai et al. proposed a new conditional entropy to evaluate the uncertainty in incomplete decision systems. Given an incomplete decision system $IDS = (U, AT \cup DT, V, f)$, $U = \{u_1, u_2, ..., u_n\}$. $B \subseteq AT$ is an attribute set; $U/D = \{Y_1, Y_2, ..., Y_m\}$. The conditional entropy (CE) of D with respect to B is defined by

$$H(D|B) = -\sum_{i=1}^{|U|}\sum_{j=1}^{m}\frac{|T_B(u_i)\cap Y_j|}{|U|}\log\frac{|T_B(u_i)\cap Y_j|}{|T_B(u_i)|}.$$

Since the conditional entropy is monotonous, and the conditional entropy is smaller, the attribute set B will be more important. We have the following definitions.

Definition 3.4 Let $\widetilde{I}_1, \widetilde{I}_2, ..., \widetilde{I}_s$ be s fuzzy incomplete information systems and $U = \{u_1, u_2, ..., u_n\}.\forall a \in \widetilde{AT}$; $U/D = \{Y_1, Y_2, ..., Y_m\}$. The uncertainty measurement of information sources of D *with respect to* $\widetilde{I}_q$ $(q = 1, 2, ..., s)$ under the attribute a is defined by

$$H_a(D|\widetilde{I}_q) = -\sum_{i=1}^{|U|}\sum_{j=1}^{m}\frac{|T_a^q(u_i)\cap Y_j|}{|U|}\log\frac{|T_a^q(u_i)\cap Y_j|}{|T_a^q(u_i)|},$$

where $T_a^q(u_i)$ is the tolerance class of information sources of D with respect to $\widetilde{I}_q$ $(q = 1, 2, ..., s)$ under the attribute a.

Since the conditional entropy of Dai et al. [35] is monotonous, $H_a(D|\widetilde{I}_q)$ $(q = 1, 2, ..., s)$ under the attribute a is also monotonous. Let $MFDT = \{\widetilde{I}_1, \widetilde{I}_2, \cdots, \widetilde{I}_s\}$ be a multi-source fuzzy decision table. For any $a \in AT$, $\widetilde{I}_k^a(k = 1, 2, \cdots, s)$ represents all the values of the kth fuzzy information system under the attribute a. The conditional entropy can be used to evaluate the importance of attributes. For the attribute a, the smaller the conditional entropy is, the more important the information source will be. We have the following Definition 3.5.

Definition 3.5 Let $\widetilde{I}_1, \widetilde{I}_2, ..., \widetilde{I}_s$ be s fuzzy incomplete information systems. The ${k_a}^{th}$ system which is most important for attribute a can be formulated as follows:

$$k_a = \mathop{\arg min}_{k \in \{1,2,\cdots,s\}} (H_a(D|\tilde{I}_k)).$$

We can create a new fuzzy incomplete information system NI, and $NI(a) = \tilde{I}^a_{k_a}, a \in \widetilde{AT}$, where $NI(a)$ represents the values of all objects under the attribute a.

The fusion process is shown in Fig. 2. There is a multi-source information box [35] $MFDT = \{\tilde{I_1}, \tilde{I_2}, \cdots, \tilde{I_s}\}$ with s fuzzy incomplete information system, and there are n objects and m attributes for each fuzzy incomplete information system $\tilde{I_i}, (i = 1, 2, \cdots, s)$. We can calculate the conditional entropy of each attribute by Definition 3.4. To find the minimum of the conditional entropy in each attribute of the values by Definition 3.5, we use different colors of rough lines to express the corresponding attributes to select the source. And the selected attribute values are integrated into a new information system.

Example 3.1 Ground object recognition is to detect the character of the ground object using remote sensing from the air, and it is based on the principle that different objects have different responses to the spectrum of the spectrum to identify various types of ground objects on the ground. The shape, size, shadow, texture, location, and color are identified by four satellites in ten different regions, and the collected data are gathered in the following four tables. Among them, $x_1 \sim x_{10}$ represent ten different regions, $a_1 \sim a_6$ represent shape, size, shadow, texture, position, and color, respectively. The attribute values represent the different wavelengths of the electromagnetic wave. Multi-source fuzzy incomplete information is created by these tables, and we use the above definition to find their similarity classes (Tables 1, 2, 3, 4).

Given an $U/D = \{\{x_1, x_2, x_6, x_8, x_9\}, \{x_3, x_4, x_5, x_7, x_{10}\}\}$. The result of multi-source conditional entropy is shown in Table 5. The smaller the conditional entropy is, the more important the information sources will be. So $\tilde{I}_1$ is most important for a_1 and a_6, $\tilde{I}_2$ is most important for a_3 and a_5, $\tilde{I}_3$ is most important for a_4, and $\tilde{I}_4$ is most important for a_2. A new fuzzy incomplete information system is established by the part of each table, $NI = (\tilde{I}_1^{a_1}, \tilde{I}_4^{a_2}, \tilde{I}_2^{a_3}, \tilde{I}_3^{a_4}, \tilde{I}_2^{a_5}, \tilde{I}_1^{a_6})$, and the result of conditional entropy fusion is shown in Table 6.

The mean fusion is the common fusion method, and we will compare the method with the conditional entropy fusion by approximation accuracy. So the results of the two fusion methods are shown in Tables 6 and 7.

Table 1 Information source I_1

U	a_1	a_2	a_3	a_4	a_5	a_6
x_1	0.5296	0.0407	0.9270	0.5556	0.2926	0.2226
x_2	0.5956	0.0411	0.5933	0.4293	0.3259	0.1593
x_3	0.4715	0.0148	0.4378	–	0.4222	0.2970
x_4	0.4822	0.0207	0.4463	0.3648	0.5556	0.2885
x_5	0.4911	–	0.4285	0.2696	0.6556	0.3307
x_6	0.7411	0.0570	0.8519	0.4463	0.2815	0.1663
x_7	0.4630	0.0215	0.4111	0.2963	–	0.2863
x_8	0.6185	0.0619	0.8333	0.4444	0.2963	0.1481
x_9	–	–	0.8241	0.4941	0.2852	0.2048
x_{10}	0.5000	0.0300	0.4296	0.3704	0.7778	0.3667

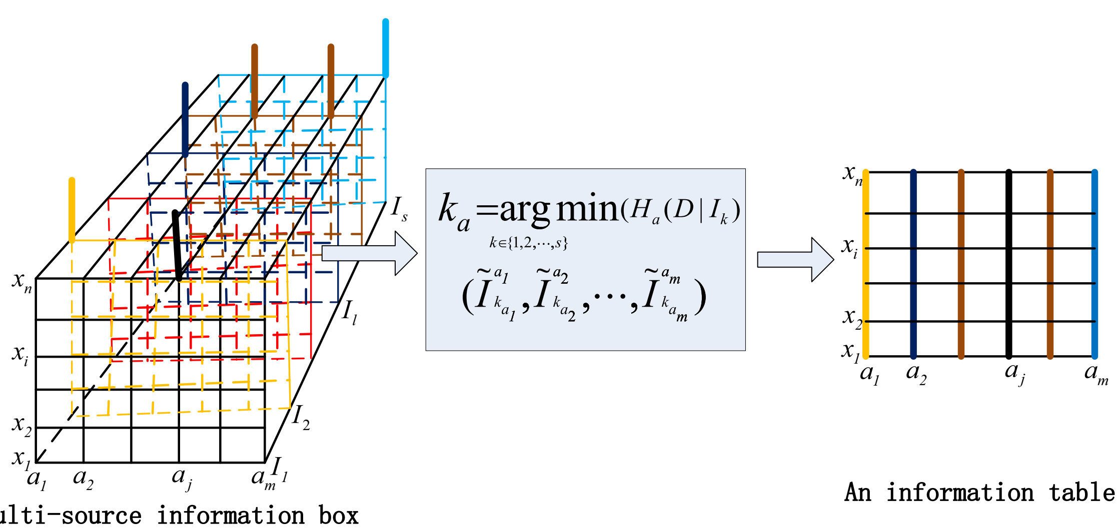

Fig. 2 Multi-source information fusion process

Table 2 Information source I_2

U	a_1	a_2	a_3	a_4	a_5	a_6
x_1	–	0.0415	0.9256	0.5548	0.2889	0.2185
x_2	0.5963	0.0407	–	0.4259	0.3222	0.1685
x_3	0.4900	0.0137	0.4463	0.3259	0.4259	0.3000
x_4	0.4733	–	0.4463	0.3667	0.5630	0.2889
x_5	0.4807	0.0233	0.4333	–	0.6481	0.3296
x_6	0.7307	0.0556	0.9989	–	0.2778	0.1674
x_7	0.4833	0.0204	–	0.2974	0.6704	0.2859
x_8	–	0.0619	0.8256	0.4481	0.3000	0.1515
x_9	0.6626	0.0493	0.8252	0.4926	0.2815	0.2037
x_{10}	0.4893	0.0293	0.4300	0.3744	0.7815	–

Table 3 Information source I_3

U	a_1	a_2	a_3	a_4	a_5	a_6
x_1	0.5189	–	0.9259	0.5559	0.2926	–
x_2	0.6111	0.0456	0.5959	0.4252	0.3259	0.1667
x_3	–	0.0156	0.4463	0.3241	0.4259	0.3000
x_4	0.4815	0.0189	0.4481	–	0.5593	0.2885
x_5	0.4837	0.0256	0.4367	0.2704	0.6519	0.3289
x_6	–	0.0622	–	0.4441	0.2778	–
x_7	0.4730	0.0193	0.4119	0.2948	0.6704	0.2852
x_8	0.6148	–	0.8196	0.4441	0.3000	0.1511
x_9	0.6437	0.0496	0.8259	0.4922	0.2852	0.2019
x_{10}	0.4944	0.0296	–	0.3711	–	0.3707

Table 4 Information source I_4

U	a_1	a_2	a_3	a_4	a_5	a_6
x_1	0.5278	0.0407	–	0.5556	0.2889	0.2222
x_2	0.6044	0.0452	0.5937	0.4222	0.3185	–
x_3	0.4937	0.0148	0.4363	0.3263	0.4259	0.3000
x_4	–	0.0185	–	0.3667	0.5556	0.2885
x_5	0.4881	–	0.4315	0.2700	–	0.3304
x_6	0.7407	0.0604	–	0.5556	0.2741	0.1667
x_7	0.4778	0.0185	0.4111	–	0.6704	0.2852
x_8	–	0.0600	0.8185	0.4452	0.3000	–
x_8	0.6370	0.0481	–	–	0.2852	0.2037
x_{10}	0.4963	0.0304	–	0.3704	0.7778	0.3696

Table 5 Conditional entropy of information sources under different attributes

$\widetilde{I}$	a_1	a_2	a_3	a_4	a_5	a_6
$\widetilde{I}_1$	2.5141	2.4615	2.4755	2.8029	2.0198	2.7224
$\widetilde{I}_2$	2.5467	2.2943	2.3583	2.8741	1.4789	3.0103
$\widetilde{I}_3$	3.0103	2.2310	2.6966	2.7224	2.0523	2.8741
$\widetilde{I}_4$	2.6553	1.9983	3.0103	2.7256	2.0198	2.9453

Table 6 Result of multi-source information fusion

U	a_1	a_2	a_3	a_4	a_5	a_6
x_1	0.5296	0.0407	0.9256	0.5559	0.2889	0.2226
x_2	0.5956	0.0452	–	0.4252	0.3222	0.1593
x_3	0.4715	0.0148	0.4463	0.3241	0.4259	0.2970
x_4	0.4822	0.0185	0.4463	–	0.5630	0.2885
x_5	0.4911	–	0.4333	0.2704	0.6481	0.3307
x_6	0.7411	0.0604	0.9989	0.4441	0.2778	0.1663
x_7	0.4630	0.0185	–	0.2948	0.6704	0.2863
x_8	0.6185	0.0600	0.8256	0.4441	0.3000	0.1481
x_9	–	0.0481	0.8252	0.4922	0.2815	0.2048
x_{10}	0.5000	0.0304	0.4300	0.3711	0.7815	0.3667

Table 7 Result of multi information sources mean fusion

U	a_1	a_2	a_3	a_4	a_5	a_6
x_1	0.5254	0.0410	0.9262	0.5555	0.2907	0.2211
x_2	0.6019	0.0431	0.5943	0.4256	0.3231	0.1648
x_3	0.4851	0.0147	0.4417	0.3254	0.4250	0.2993
x_4	0.4790	0.0194	0.4469	0.3660	0.5583	0.2886
x_5	0.4859	0.0244	0.4325	0.2700	0.6519	0.3299
x_6	0.7375	0.0588	0.9254	0.4820	0.2778	0.1668
x_7	0.4743	0.0199	0.4114	0.2962	0.6704	0.2856
x_8	0.6167	0.0612	0.8243	0.4455	0.2991	0.1502
x_9	0.6478	0.0490	0.8251	0.4930	0.2843	0.2035
x_{10}	0.4950	0.0298	0.4298	0.3716	0.7790	0.3690

Through Tables 6 and 7, we can compute the approximation accuracy of the results of two kinds of fusion and compare their approximation accuracy (Table 8).

By comparing the approximate accuracy, it is easy to see that our fusion method has a higher precision than the mean value fusion method. In real life, a large amount of data are needed to be fused, which has very high requirements on the precision and correct rate for a considerable part of the information. The fusion method proposed in this paper can be more accurate than the common method, and it has more practical significance. In order to better understand the method proposed in this paper, we design the conditional entropy fusion algorithm (Algorithm 1) and analyze computational complexity and time complexity. In order to make our algorithm easier to understand, we designed the algorithm flow chart in Fig. 3.

Algorithm 1: An algorithm for conditional entropy fusion

Input : A multi-source fuzzy incomplete information system $MFI = \{\tilde{I}_1, \tilde{I}_2, \cdots, \tilde{I}_s\}$, a classification $U/D = \{Y_1, Y_2, \cdots, Y_m\}$;
Output : A new information table.

1 **begin**
2   **for** $k = 1 : s$ **do**
3   **for** *each* $a \in AT$ **do**
4   **for** *each* $x_i \in U$ **do**
        compute: $T_a^k(x_i)$; // compute all $T_a^k(x_i)$, for any $x_i \in U$ under attribute a;
5   **end**
6       $HCE \leftarrow 0$;
7   **for** $i = 1 : |U|$ **do**
8   **for** $j = 1 : m$ **do**
9   **if** $|T_a^k(x_i \cap Y_j| > 0$ **then**
10             $HCE \leftarrow HCE - \frac{|T_a^k(x_i \cap Y_j|}{|U|} log \frac{|T_a^k(x_i \cap Y_j|}{|T_a^k(x-i|}$;
11   **end**
12   **end**
13   **end**
14       $H_a(D|\tilde{I}_k) \leftarrow HCE$; // record CE under attribute a with the k^{th} information source;
15   **end**
16   **end**
17   **for** *each* $a \in AT$ **do**
18     $minCE \leftarrow +\infty$;
19   **for** $k = 1 : s$ **do**
20   **if** $H_a(D|\tilde{I}_k) < minCE$ **then**
21         $minCE \leftarrow H_a(D|\tilde{I}_k)$;
22         $k_a \leftarrow k$;
23   **end**
24   **end**
25   **end**
26   **return**: $(\tilde{I}_{k_{a_1}}^{a_1}, \tilde{I}_{k_{a_2}}^{a_2}), \cdots, \tilde{I}_{k_{a_{|AT|}}}^{a_{|AT|}})$.
27 **end**

The given algorithm (Algorithm 1) is a new approach for multi-source information fusion. Its provides a better approximation accuracy than the mean fusion method from the result of Example 3.1. First, we can calculate all the similarity classes $T_a^q(x)$, for any $x \in U$ under attribute a. Then, the conditional entropy $H_a(D|\widetilde{I_q})$ is computed with the q^{th} information source under attribute a. Finally, the minimum of the conditional entropy of the information source is selected under the attribute a and using these results splice into a new table. The computational complexity of Algorithm 1 is shown in Table 9.

Next, we can analyze the time complexity of Algorithm 1 step by step. For Steps 4 and 5, the time complexity to calculate the all $T_a^q(x)$ is denoted by $t_1 = O(|U|^2)$, for any $x \in U$ under attribute a. Steps 6 to 14 compute the conditional entropy with the q information source under attribute a, and the time complexity to finish Steps 6 to 14 is $t_2 = O(|U| \times m^2)$. Steps 17 to 26 find out the minimum of the conditional entropy of the corresponding source for any $a \in \widetilde{AT}$, and the time complexity to finish Steps 17 to 26 is $t_3 = O(|AT| \times s) + O(|U| \times |AT|)$.

4 Comparisons of Fusion Methods

In this section, we will study the differences between the two fusion methods, in order to highlight the differences between the methods proposed in the literature and in this paper. In this paper, the fusion process of the two fusion methods is given in the form of figure.

Case 1 Conditional entropy fusion method

The conditional entropy fusion process is shown in Fig. 4. The conditional entropy is used to calculate the fusion of multi-source information. in the fusion process, firstly the conditional entropy of each source is calculated,

Table 8 Approximation accuracies of two kinds of fusion methods

	Multi-source fusion	Mean fusion
Approximation accuracy	0.6667	0.5385

and then the minimum value of each source in a given attribute is found by calculating the conditional entropy; finally, the minimum value corresponding to each of the attributes of each source is drawn out to construct a new information table.

Case 2 D-S evidence theory fusion method

The conditional entropy fusion process is shown in Fig. 5. The D-S evidence theory is used to calculate the fusion of multi source information. In the fusion process, first the basic probability function m_i , trust function Bel_i, and likelihood function Pl_i of each sensor monitoring data are calculated, and then, using D-S evidence theory synthesis rules, all the evidence in the joint function of the basic probability assignment function, the trust function, and the likelihood function is calculated; finally, according to the decision rules, the maximum support is selected as the fusion result (Table 10).

5 Experimental Evaluation

In this section, in order to further illustrate the correctness of conclusion of the above example, we conduct a series of experiments to demonstrate that the approximate precision of most of the conditional entropy fusion is higher than that of the average value fusion based on standard datasets from the machine learning data repository, University of California at Irvine(http://archive.ics.uci.edu/ml/datasets.html), namely “Statlog(Vehicle Silhouettes),” “Wine Quality-red,” “Wine Quality-white,” “Combined Cycle Power Plant,” “EEG Eye State,” “PPPTS,” “UJIIndoorLocMag,” “Default of credit card clients,” and “KEGG Metabolic Relation Network (Directed).” This experimental computing program is running on a personal computer with the following hardware and software as in Table 11.

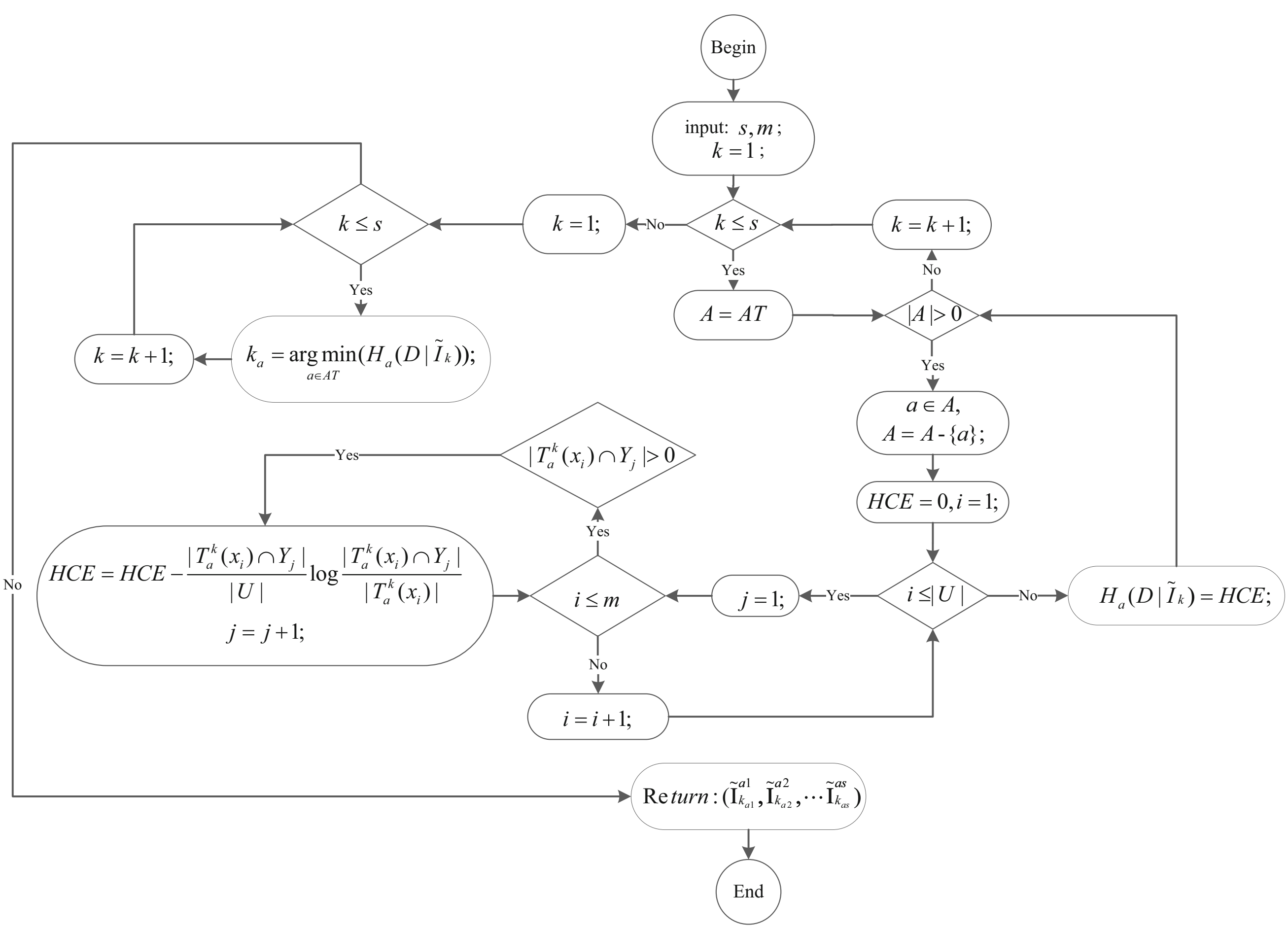

Fig. 3 Flow diagram of Algorithm 1

Table 9 Computational complexity of Algorithm 1

Steps 4–5	$O(\|U\|^2)$
Steps 6–14	$O(\|U\| \times m^2)$
Steps 1–16	$O(s \times \|AT\| \times (\|U\|^2 + \|U\| \times m^2))$
Steps 17–25	$O(\|AT\| \times s)$
Step 26	$O(\|U\| \times \|AT\|)$
Total	$O(s \times \|AT\| \times (\|U\|^2 + \|U\| \times m^2) + \|AT\| \times s + \|U\| \times \|AT\|)$

It is very difficult to directly download fuzzy incomplete data from the network, so a method is proposed to obtain multi-source fuzzy incomplete data. Firstly, in order to obtain fuzzy data, original data in each column were divided by the maximum value of the column. Then, a multi-fuzzy decision table is constructed by adding Gauss noise and random noise to the fuzzy information table. Finally, in order to obtain multi-source fuzzy incomplete datasets, the multi-fuzzy decision tables are used as the multi-source fuzzy incomplete datasets after they delete randomly some data.

Let $MFDT = \{\tilde{I_1}, \tilde{I_2}, \cdots, \tilde{I_s}\}$ be a multi-source fuzzy incomplete decision table constructed by a original fuzzy information table $\tilde{I}$. First s numbers $(g_1, g_2, \cdots, g_s)$ are generated and these numbers obey the $N(0, \sigma)$ distribution, where σ is the standard deviation. The method of adding Gauss noise is defined as follows:

$$\tilde{I}_i(x,a) = \begin{cases} \tilde{I}(x,a) + g_i & \textit{if } (0 \leq \tilde{I}(x,a) + g_i \leq 1) \\ \tilde{I}(x,a) & \textit{else} \end{cases},$$

where $\tilde{I}(x,a)$ represents the value of object x under attribute a in fuzzy information table and $\tilde{I}_i(x,a)$ represents object x under attribute a in the i^{th} fuzzy information source.

Then s random numbers $(e_1, e_2, \cdots, e_s)$ are generated and these numbers are between $-e$ and e, where e is the random error threshold. The method of adding random noise is described in the following.

$$\tilde{I}_i(x,a) = \begin{cases} \tilde{I}(x,a) + e_i & \textit{if } (0 \leq \tilde{I}(x,a) + e_i \leq 1) \\ \tilde{I}(x,a) & \textit{else} \end{cases},$$

where $\tilde{I}(x,a)$ represents the value of object x under attribute a in fuzzy information table and $\tilde{I}_i(x,a)$ represents object x under attribute a in the i^{th} fuzzy information source.

Next, 40 % objects are randomly selected from the fuzzy information table $\tilde{I}$ and then Gauss noise is added to these objects. 20 % objects are randomly selected from the rest of the table and random noise is added. And we delete randomly some data for the fuzzy information table $\tilde{I}$. Finally, $MFDT = \{\tilde{I_1}, \tilde{I_2}, \cdots, \tilde{I_s}\}$ can be achieved.

In different science fields, the standard deviation of Gauss noise and random error threshold of random noise may be different. In this paper, we will conduct the experiments 20 times for each dataset and set the standard

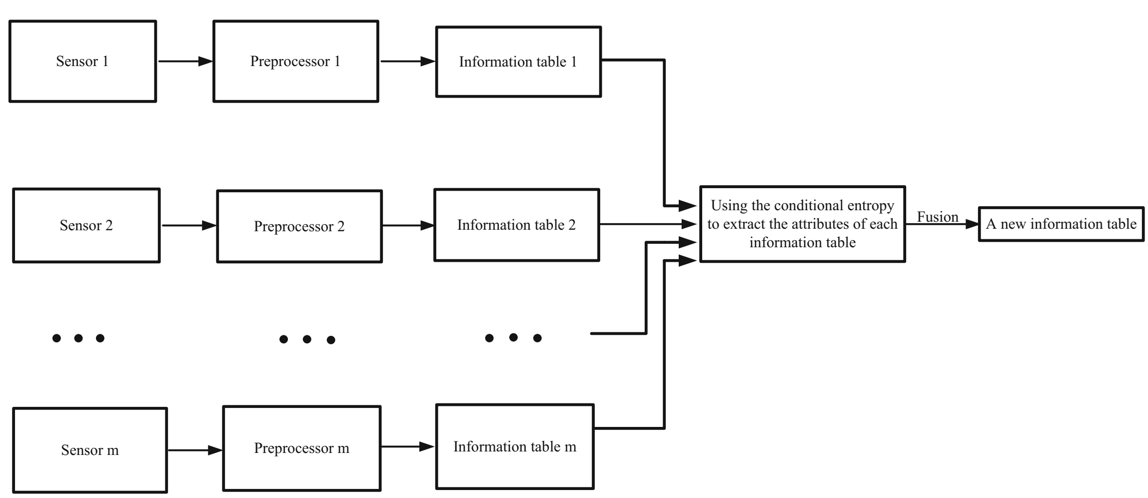

Fig. 4 Conditional entropy fusion process

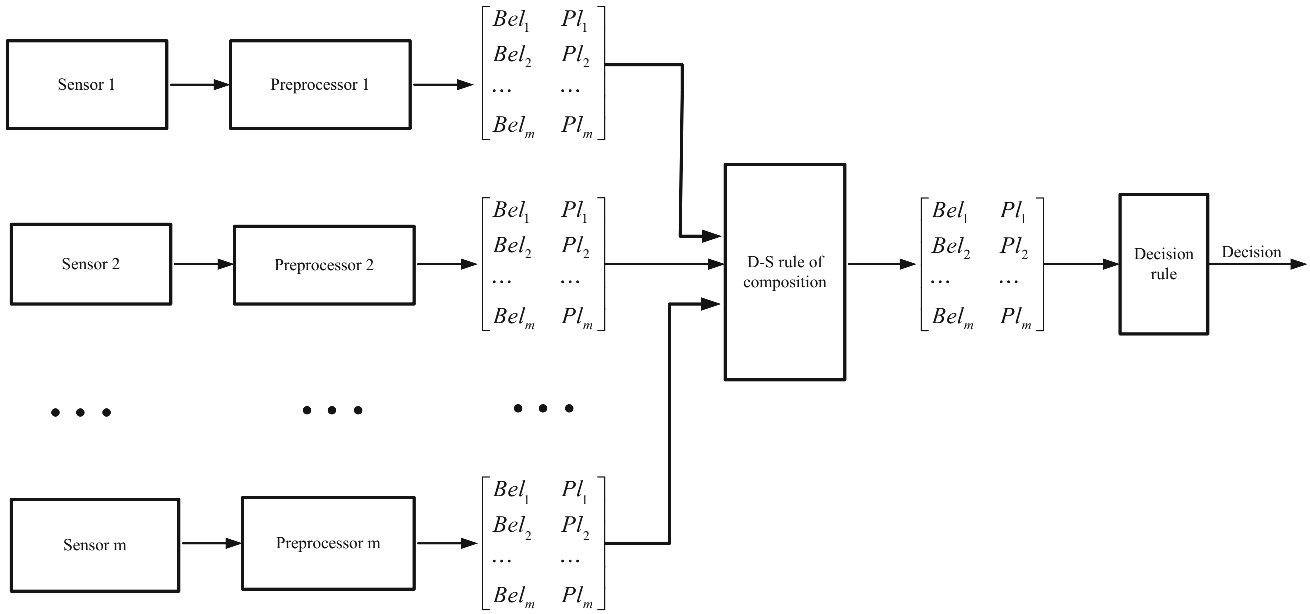

Fig. 5 D-S evidence theory fusion process

Table 10 Experiment datasets

No.	Dataset name	Abbreviation	Objects	Attributes	Decision classes	Number of sources	Elements of MDT
1	Statlog (vehicle silhouettes)	S(VS)	846	19	4	10	160,740
2	Wine quality-red	WQ-r	1,599	12	6	10	191,880
3	Wine quality-white	WQ-w	4,898	12	7	10	587,760
4	Combined cycle power plant	CCPP	9,568	6	6	10	574,080
5	EEG eye state	EES	14,980	15	2	10	2,247,000
6	PPPTS	PPP	25,000	11	6	10	2,750,000
7	Default of credit card clients	DCCC	30,000	24	8	10	7,200,000
8	UJIIndoorLoc-Mag	U-M	40,000	13	10	10	5,200,000
9	KEGG metabolic relation network	KMRN	53,414	24	7	10	12,819,360

Table 11 Description of experiment environment

Name	Model	Parameters
CPU	$Intel \quad i7-5500U$	2.40 GHz
Memory	SamsungDDR3	8 GB; 1067 MHz
HardDisk	WestData	1 TB
System	Windows10	64 bit
Platform	$C++$	6.0

deviation σ and random error threshold e from 0 to 2 and each time increase by 0.05. Under the CE fusion and mean fusion, the approximation accuracy of U/D for each dataset can be displayed in Table 12 and Figs. 4, 5, 6, 7, 8, 9, 10, and 11. CE and M stand for CE fusion and mean fusion, respectively.

From Fig. 4, we can find that although accuracy of the first decision class of conditional entropy fusion is much lower than the average fusion accuracy, the accuracies of the other five decision classes of conditional entropy of fusion are mostly higher than the accuracy of the mean value fusion as the noise continues to increase. And the approximation accuracies of all decision classes of conditional entropy fusion are much higher than that of the mean value fusion. From Fig. 5, we find that the fusion accuracy of conditional entropy of each decision class is much higher than that of the mean value fusion in addition to the accuracy of the first and seventh decision classes with several conditional entropy fusion equal to the mean value fusion, and the approximation accuracies of all decision

Table 12 Approximation accuracies of CE fusion and mean fusion for each dataset

WQ-r		WQ-w		CCPP		EES		PPP		Dccc		U-M		KMRN	
CE	M.	CE	M.	CE	M.	CE	M.	CE	M.	CE	M.	CE	M.	CE	M.
0.2958	0.174	0.1679	0.0108	0.0544	0.0275	0.2108	0.0001	0.0377	0.0025	0.1586	0.012	0.1452	0.0357	0.1768	0.0232
0.3232	0.2248	0.1983	0.0194	0.0672	0.031	0.3784	0.0002	0.075	0.0025	0.1967	0.0234	0.1456	0.0543	0.2431	0.0198
0.3321	0.2509	0.2157	0.0256	0.0853	0.0344	0.4157	0.0003	0.1162	0.0023	0.3578	0.0397	0.0564	0.0343	0.2234	0.0276
0.3298	0.278	0.22	0.035	0.0902	0.0379	0.4223	0.0005	0.1452	0.0022	0.238	0.0343	0.0653	0.0365	0.2245	0.0453
0.3412	0.2882	0.22	0.0469	0.1033	0.0375	0.4242	0.0007	0.1639	0.0017	0.2167	0.0326	0.1342	0.0276	0.2278	0.0563
0.3351	0.2915	0.2223	0.0505	0.1091	0.0396	0.4249	0.0015	0.1746	0.0019	0.2348	0.0568	0.1087	0.0254	0.2256	0.0543
0.3385	0.3015	0.2212	0.0567	0.1131	0.0433	0.4249	0.002	0.1767	0.0017	0.2212	0.0531	0.1254	0.0427	0.2266	0.0657
0.3336	0.2969	0.2202	0.0572	0.107	0.0423	0.4228	0.0023	0.1773	0.0014	0.21	0.0542	0.1457	0.0463	0.3452	0.0653
0.3344	0.3156	0.2191	0.0599	0.1062	0.0443	0.4226	0.0017	0.1694	0.0016	0.2134	0.0457	0.1078	0.0452	0.2246	0.0683
0.3313	0.3008	0.2133	0.0628	0.098	0.0463	0.4184	0.0023	0.1618	0.0016	0.34	0.0612	0.1235	0.0478	0.2765	0.0754
0.3313	0.305	0.2123	0.0604	0.0981	0.0459	0.4149	0.0022	0.1535	0.0017	0.2321	0.0406	0.1356	0.0578	0.2231	0.0653
0.3232	0.301	0.2105	0.0632	0.0946	0.0464	0.4082	0.0016	0.1437	0.0018	0.346	0.0542	0.0958	0.0458	0.2134	0.0627
0.3264	0.2862	0.2007	0.0587	0.0887	0.0484	0.4033	0.002	0.1367	0.0018	0.435	0.0552	0.1865	0.0437	0.2754	0.0598
0.319	0.301	0.1992	0.058	0.0877	0.0486	0.3926	0.0016	0.1262	0.0017	0.1884	0.0325	0.0946	0.0479	0.2329	0.0576
0.3036	0.3041	0.192	0.0541	0.0835	0.0517	0.3872	0.0014	0.119	0.0018	0.213	0.0534	0.0764	0.0523	0.2012	0.0531
0.3126	0.2961	0.1882	0.0556	0.0781	0.0518	0.377	0.0015	0.1099	0.0019	0.1882	0.0546	0.0886	0.0476	0.1943	0.0562
0.3078	0.3033	0.1841	0.0541	0.0793	0.0522	0.3679	0.0008	0.1012	0.0019	0.1831	0.0744	0.0675	0.0542	0.1856	0.0572
0.297	0.2987	0.1813	0.0513	0.0754	0.0507	0.3536	0.0009	0.1	0.0019	0.1954	0.0627	0.0764	0.0584	0.1824	0.0502
0.2992	0.2977	0.1736	0.0524	0.0711	0.0491	0.3428	0.0009	0.0907	0.0017	0.1637	0.0456	0.0875	0.0459	0.1742	0.0531
0.2908	0.3048	0.1612	0.0489	0.0697	0.0511	0.3297	0.0007	0.087	0.0019	0.1432	0.0764	0.0675	0.0581	0.1954	0.0587

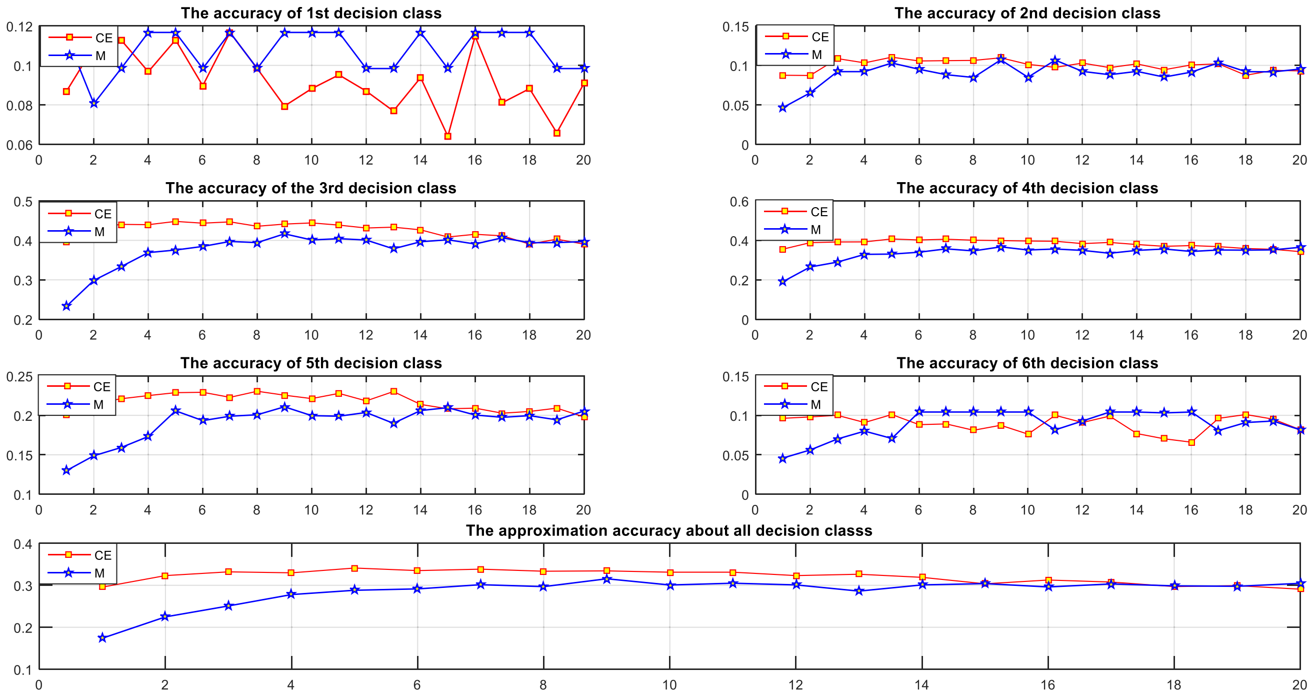

Fig. 6 Approximation accuracies of decision classes in the dataset WQ-r

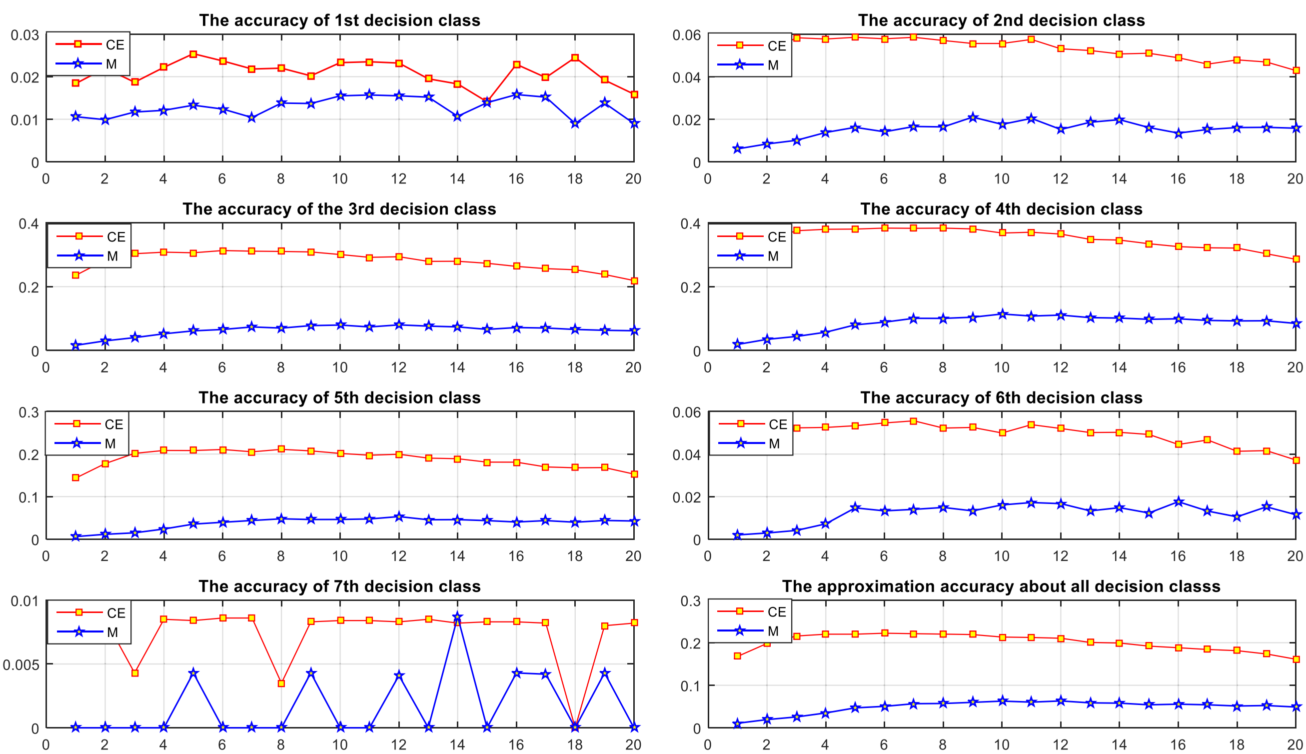

Fig. 7 Approximation accuracies of decision classes in the dataset WQ-w

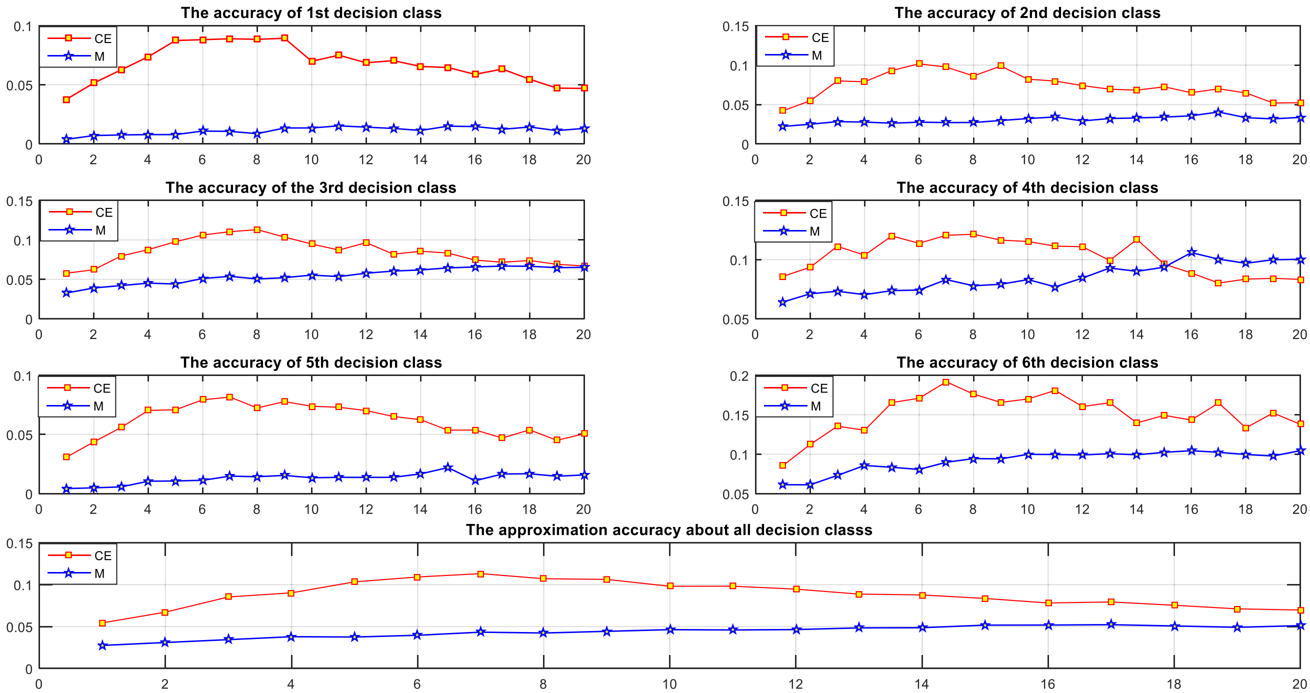

Fig. 8 Approximation accuracies of decision classes in the dataset CCPP

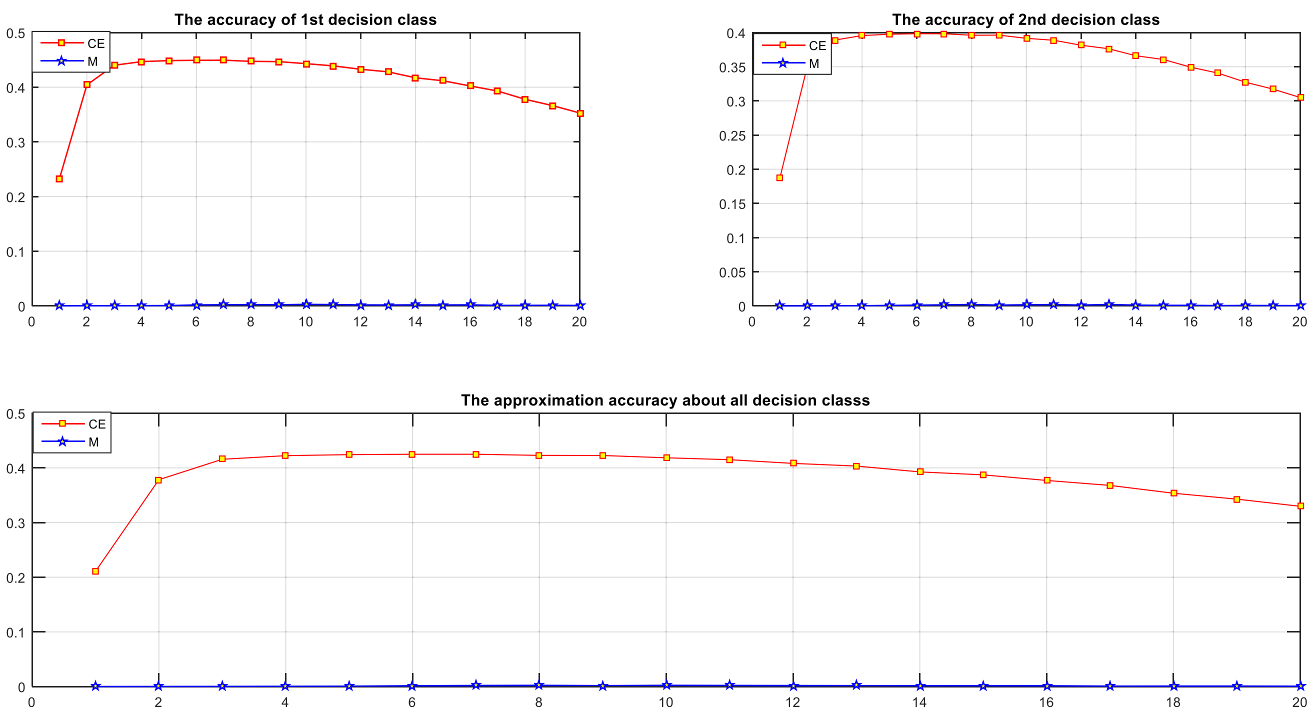

Fig. 9 Approximation accuracies of decision classes in the dataset EES

classes for the conditional entropy fusion are higher than that of the mean value fusion when the noise continues to increase. From Fig. 6, we can see that the accuracy of conditional entropy fusion for each decision class is much higher than that of the mean value fusion in addition to the fourth decision class with five conditional entropy fusion

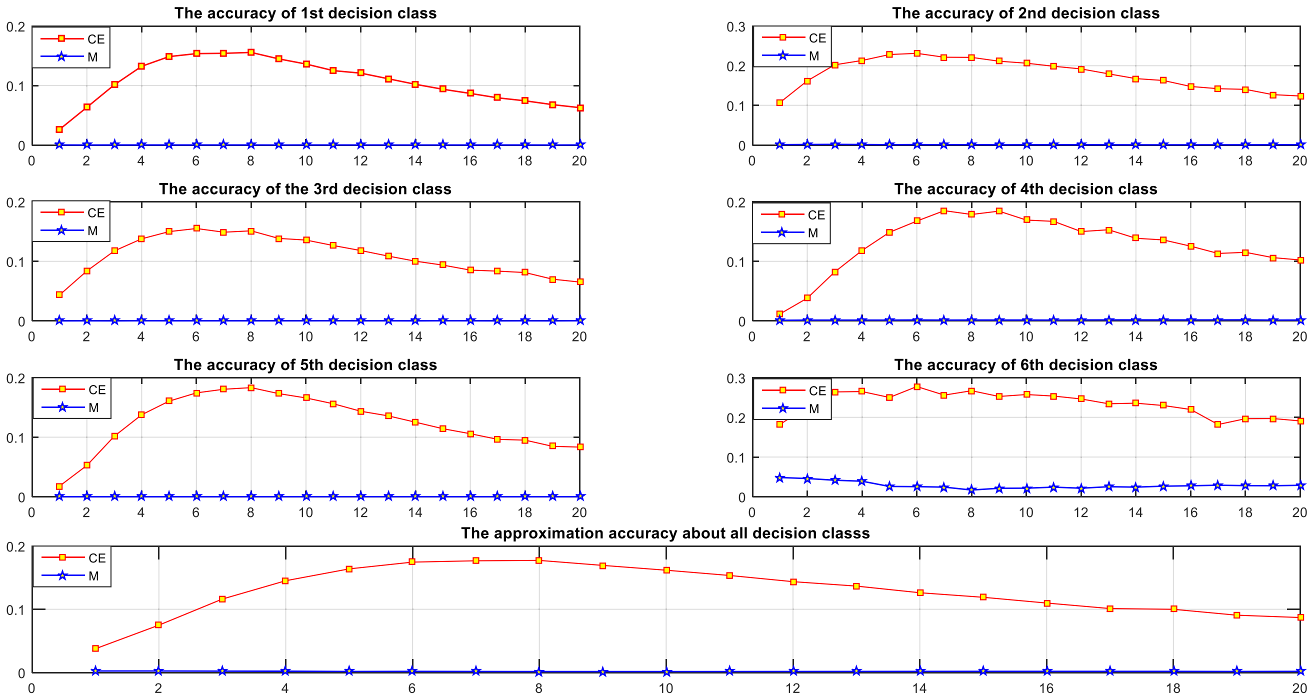

Fig. 10 Approximation accuracies of decision classes in the dataset PPP

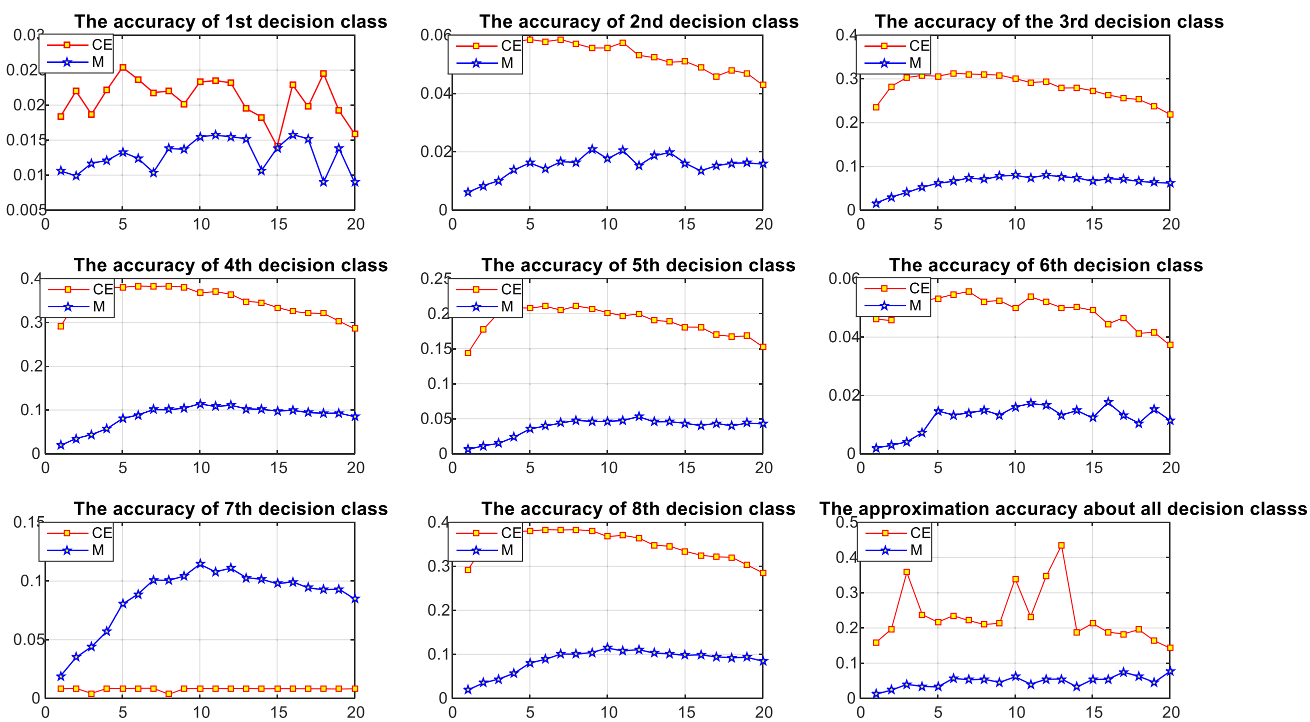

Fig. 11 Approximation accuracies of decision classes in the dataset Dccc

accuracy below the mean value fusion, and the approximation accuracies of all decision classes in the conditional entropy fusion are higher than that of the mean value fusion. From Figs. 7, 8, and 9, we find that as the noise continues to increase, the accuracy of conditional entropy fusion of each decision class is higher than that of mean

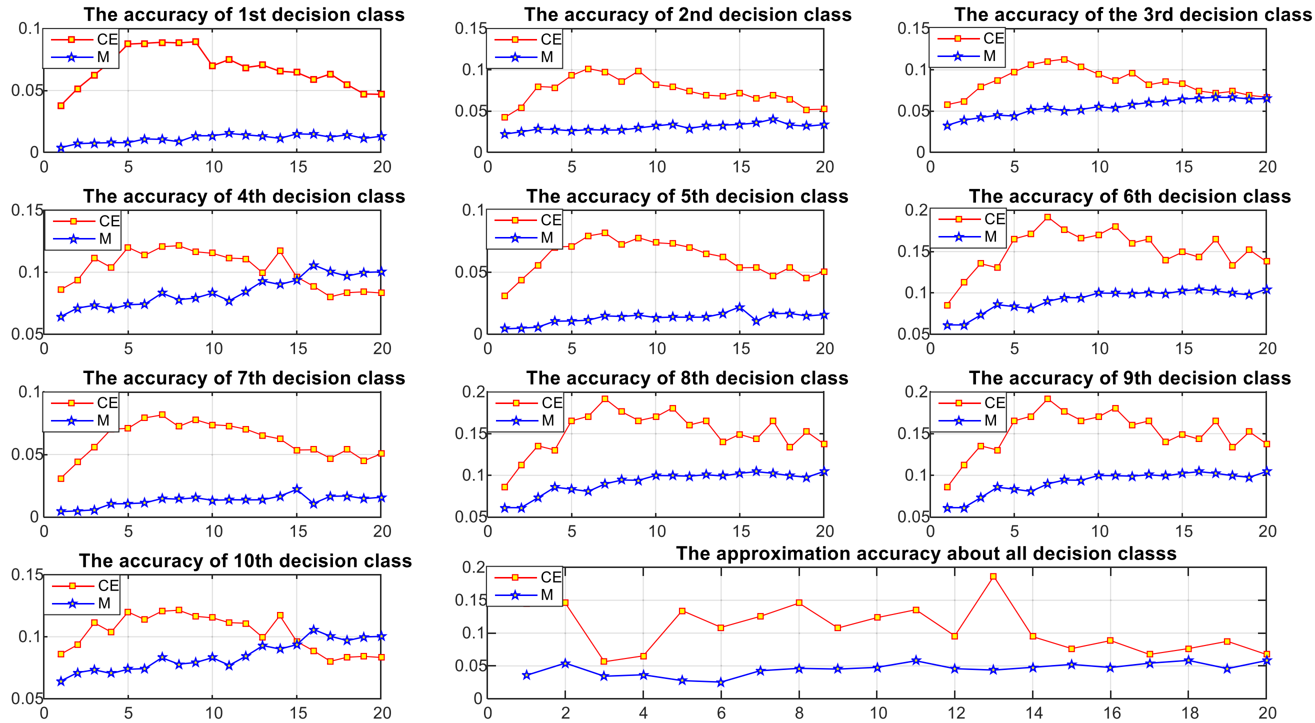

Fig. 12 Approximation accuracies of decision classes in the dataset U-M

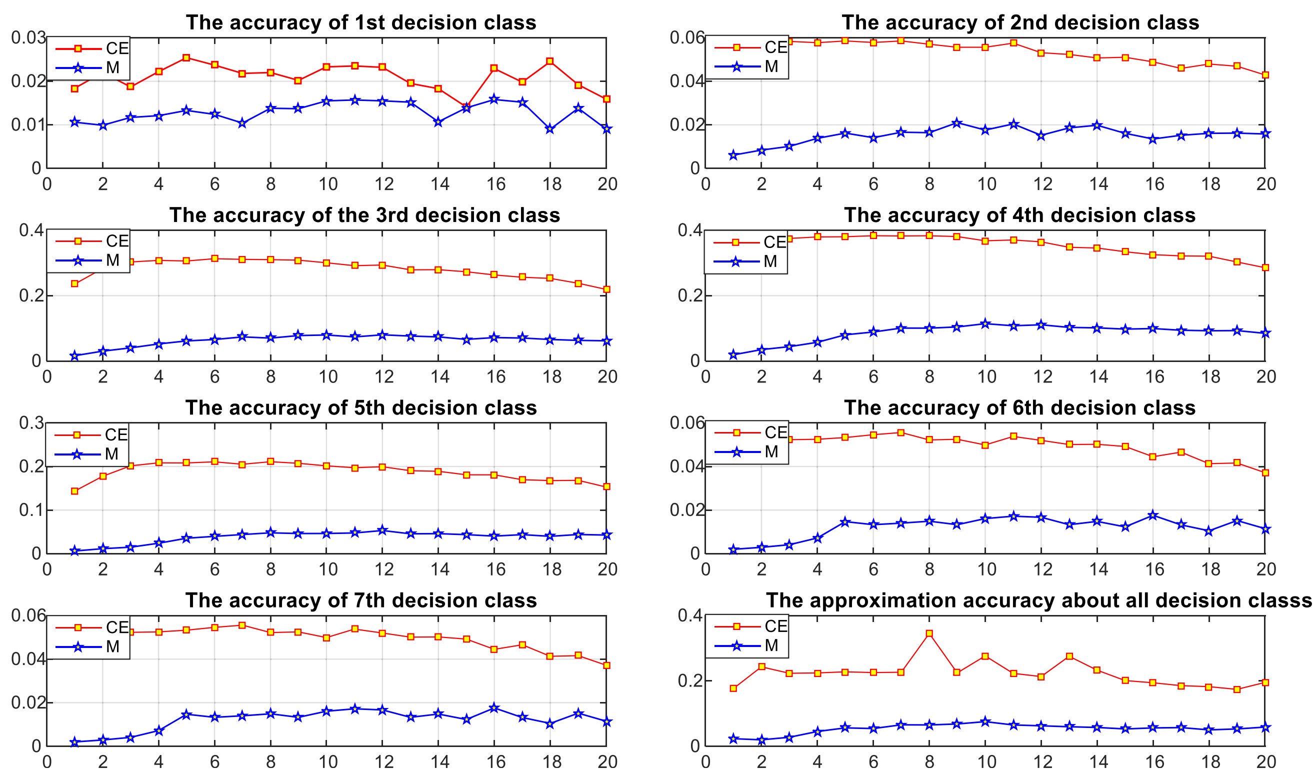

Fig. 13 Approximation accuracies of decision classes in the dataset KMRN

value fusion, and the approximation accuracies of all decision classes in the conditional entropy fusion are also higher than that of the mean value fusion. From Fig. 10, the accuracy of conditional entropy fusion for each decision class is much higher than that of the mean value fusion in addition to the fourth and the tenth decision classes with several conditional entropy fusion accuracies lower than that of mean value fusion with the increase of noise. From Fig. 11, with the constant increase in noise, the accuracy of the conditional entropy fusion for each decision class is much higher than the average value of fusion except that the first decision class has an accuracy of conditional entropy fusion equal to that of the mean value fusion.

As is vividly described in Figs. 6,7, 8, 9, 10, 11, 12, 13 and Table 12, we can see that when the noise is small, in most cases, the approximation accuracy of CE fusion is slightly higher than the approximation accuracy of mean fusion. In a certain range, with the increase of noise, the approximation accuracy of CE fusion is much better than that of mean fusion. By observing the approximation accuracies of the extension of concepts of CE fusion and mean fusion about six datasets, we find that in most cases the approximation accuracy of CE fusion is higher than the approximation accuracy of mean fusion. In a certain range, with the increase of noise, the accuracies of the extension of concepts of CE fusion and mean fusion have an uptrend, but they are not strictly monotonic.

6 Conclusions

At the present day, every people can get huge amounts of data and information, these data and information are not all clear and complete, how the information fusion is seem very important. An effective multi-source fuzzy incomplete information fusion method based on information entropy is proposed in this paper. By this method, we can merge multiple fuzzy incomplete sources into an information table. By comparing the approximation accuracy, we can get the information entropy fusion method better than the average value fusion method. In the last part of this paper, experiments on UCI datasets demonstrate that the proposed method is simple compared to the average value fusion method and verify the effectiveness of this method. The method in this paper can be effective for the fusion of multi-source fuzzy incomplete information systems. This method further consummates the situation of information fusion in multi-source environment.

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Weihua Xu received his B.Sc. degree in Mathematics from Yanbei Normal University, Datong, China, in 2001, M.Sc. degree in Mathematics from Guangxi University, Nanning, China, in 2004, and Ph.D. degree in Applied Mathematics from Xi'an Jiaotong University, Xi'an China, in 2007. He is a senior member of the International Society of Rough Sets. Now, he is currently a Professor and Vice-Dean of the School of Mathematics and Statistics in Chongqing University of Technology. He has published 2 monographs and more than 70 articles in international journals and book chapters. His current research interests include approximate reasoning, fuzzy sets, rough sets, concept lattices, and cognitive computing. He serves in the editorial boards of several international journals.

Mengmeng Li received his B.Sc. degree from School of Mathematics and Information, Xinyang University, Xinyang, China, in 2014. He is currently pursuing his M.Sc. degree in School of Mathematics and Statistics, Chongqing University of Technology. His current research interests include fuzzy sets, rough sets, concept lattice, and granular computing.

Xizhao Wang , Ph.D., is a Professor, an IEEE Fellow, and Editor-in-Chief of Springer Journal Machine Learning and Cybernetics. He received his doctor degree in Computer Science from Harbin Institute of Technology in September 1998. From 1998 to 2001, he worked at the Department of Computing in Hong Kong Polytechnic University as a research fellow. From October 2000 to March 2014, he served in Hebei University as the Dean of school of Mathematics and Computer Sciences. From October 2007 to March 2014, he served as the founding Director of Key Lab on Machine Learning and Computational Intelligence in Hebei Province, China. From September to November in 2013, he was a visiting professor of Canada Simon Fraser University. From December 2013 to January 2014, he was a visiting professor of University of Alberta in Canada. From July to September of 2014, he was a visiting professor of Australia New South Wales University at Canberra. Since March 2014, Prof. Wang has been working in College of Computer Science and Software Engineering in Shenzhen University as a professor and the vice director of Big Data Institute.