Architecture selection for networks trained with extreme learning machine using localized generalization error model

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A B S T R A C T
The initial localized generalization error model (LGEM) aims to find an upper bound of error between a target function and a radial basis function neural network (RBFNN) within a neighborhood of the training samples. The contribution of LGEM can be briefly described as that the generalization error is less than or equal to the summation of three terms: training error, stochastic sensitivity measure (SSM), and a constant. This paper extends the initial LGEM to a new LGEM model for single-hidden layer feed-forward neural networks (SLFNs) trained with extreme learning machine (ELM) which is a type of new training algorithms without iterations. The development of this extended LGEM can provide some useful guidelines for improving the generalization ability of SLFNs trained with ELM. An algorithm for architecture selection of the SLFNs is also proposed based on the extended LGEM. Experimental results on a number of benchmark data sets show that an approximately optimal architecture in terms of number of neurons of a SLFN can be found using our method. Furthermore, the experimental results on eleven UCI data sets show that the proposed method is effective and efficient.

1. Introduction

The generalization ability of a trained feed-forward neural network, which refers to the accuracy of the trained network predicting the classes of unseen instances, is the most important index for designing and training a neural network [1–9]. How to improve the generalization ability is the central part of learning feed-forward neural networks. The generalization ability can be equivalently described as generalization error. Given a data set, there are two frequently-used approaches from the literature to measuring and estimating generalization error. One is the cross validation [1] and the other is the error bound model [5–8]. The cross validation is based on a split of the given data set, or generally, based on a K-fold partition of the given data set, where K is an integer bigger than or equal to 2. Specifically, the given data set is first divided to K disjoint subsets, and then the union of any (K − 1) subsets is considered as a training set, and the remaining subset is the testing set. Correspondingly, K neural network classifiers are generated and the average of testing errors of the K classifiers on the remaining subset is referred as to an estimation of the generalization error, where the testing error of a classifier is defined as the number of testing instances wrongly predicted by the classifier. The K-fold cross validation is experimentally driven and easy-implementation, but it is time consuming. The error bound model for a trained neural network is to estimate the generalization error by finding an upper bound of the difference between the target function and the trained neural network on the entire input space. The main advantage of error bound models is that the error bound can be definitely suitable for all unseen instances and the main disadvantages are the difficulty of mathematical derivation in finding the bound and the looseness of the derived upper bound. Recently, based on the stochastic sensitivity analysis of the radial basis function neural network (RBFNN), a localized generalization error model (LGEM) is proposed in [10], which gave an upper bound on the generalization error for unseen samples located within neighborhoods of the training samples. Another important index for training a neural network is the complexity of training algorithm. One of the old frequently-used algorithms for training a feed-forward neural network is the gradient-based back-propagation (BP) algorithm. Even if its performance is often claimed in textbooks or references to be better, it is criticized as time-consuming of iteration and it often reaches local minimum. Recently, a new training algorithm for single-hidden layer feed-forward neural networks (SLFNs) was firstly proposed in [11]. It is referred to as Extreme Learning Machine (ELM) which performs on a SLFN. The ELM’s main features include the following points:

(1) ELM does not need iteration to tune the weights and so it is not time-consuming.

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(2) The weights between input layer and hidden layer are randomly generated.

(3) The weights between hidden layer and output are determined by solving the system of generalized linear equations.

(4) There is no general conclusion for the generalization ability of SLFNs trained by ELM.

The related references [11–18, 25–29–31] can be found from journals and conference proceedings. More recently, a survey paper on ELM, which completely introduces ELM’s historical development, newest advances, key issues, and main advantages and disadvantages, has been published in Journal of Machine Learning and Cybernetics [16]. Since there is still no general result for the generalization ability of SLFNs trained by ELM (point (4) mentioned above), based on the error bound model given in [10] for RBFNNs, this paper makes an attempt to establish an upper error bound model for a SLFN trained by ELM. A main concept for deriving the upper error bound is called Q-neighbors. It is expected that the establishment of the upper error bound model can provide some useful guidelines for clarifying the generalization ability of SLFNs trained by ELM. Moreover, the upper bound model here has potential applications to topics such as feature selection and architecture selection of networks.

This paper is organized as follows. Section 1 is the introduction. In Section 2, the ELM for SLFNs and LGEM for RBFNNs are briefly reviewed. In Section 3, the LGEM for SLFNs trained with ELM is derived in detail and its application to architecture selection is presented. In Section 4, a number of numerical experiments using this newly proposed model are conducted and some experimental results and remarks are showed there. Section 5 concludes this paper.

2. Brief review of extreme learning machine and localized generalization error model

In this section, we briefly review the basic concepts and methods of extreme learning machine and localized generalization error model.

2.1. Extreme learning machine (ELM)

The ELM algorithm was proposed by Huang for single-hidden layer feed-forward neural networks (SLFNs). According to Theorem 2.1 in [11], the input weights and biases do not need to be adjusted. It is possible to analytically determine the output weights by finding the least-square solution. The neural network is obtained after a few steps with very low computational cost.

Given a training data set, \( \{ (x_i, t_i) \mid x_i \in \mathbb{R}^n, t_i \in \mathbb{R}^m, i = 1, \ldots, N \} \), where \( x_i \) is an \( n \times 1 \) input vector and \( t_i \) is a \( m \times 1 \) target vector, a SLFN with \( M \) hidden nodes is formulated as

\[
\hat{f}_Q(x_i) = \sum_{j=1}^{M} \beta_j g(w_j \cdot x_i + b_j) = t_i \quad (1 \leq i \leq N)
\]

where \( w_j = [w_{j1}, w_{j2}, \ldots, w_{jn}]^T \) is the weight vector connecting the \( j \)-th hidden node with the input nodes, \( \beta_j = [\beta_{j1}, \beta_{j2}, \ldots, \beta_{jm}]^T \) is the weight vector connecting the \( j \)-th hidden node with the output nodes, and \( b_j \) is the threshold of the \( j \)-th hidden node. Eq. (1) can be written in a more compact format as

\[
H \beta = T
\]

where

\[
H = \begin{bmatrix}
g(w_1 \cdot x_1 + b_1) & \ldots & g(w_M \cdot x_1 + b_M) \\
g(w_1 \cdot x_2 + b_1) & \ldots & g(w_M \cdot x_2 + b_M) \\
\vdots & \ddots & \vdots \\
g(w_1 \cdot x_N + b_1) & \ldots & g(w_M \cdot x_N + b_M)
\end{bmatrix}_{N \times M}
\]

\[
\beta = \begin{bmatrix}
\beta_1 \\
\beta_2 \\
\vdots \\
\beta_M
\end{bmatrix}_{M \times m}
\]

and

\[
T = \begin{bmatrix}
t_1 \\
t_2 \\
\vdots \\
t_N
\end{bmatrix}_{N \times m}
\]

\( H \) is the hidden layer output matrix of the network [12], where the \( j \)-th column of \( H \) is the \( j \)-th hidden node's output vector with respect to inputs \( x_1, x_2, \ldots, x_N \), and the \( i \)-th row of \( H \) is the output vector of the hidden layer with respect to input \( x_i \). If the number of hidden nodes is equal to the number of distinct training samples, the matrix \( H \) is square and invertible, and SLFNs can approximate these training samples with zero error. But generally, the number of hidden nodes is much less than the number of training samples. Therefore, \( H \) is a non-square matrix and we cannot expect an exact solution of the system (2). Fortunately, it has been proved in [13,14] that SLFNs with random hidden nodes have the universal approximation capability and the hidden nodes could be randomly generated. According to the definition of the Moore-Penrose generalized inverse, the smallest norm least-squares solution of (2) is given in [11]:

\[
\hat{\beta} = H^+ T
\]

where \( H^+ \) is the Moore-Penrose generalized inverse of matrix \( H^T \).

In the following, the ELM Algorithm [11] is introduced.

ELM Algorithm: Given a training data set \( \{ (x_i, t_i) | x_i \in \mathbb{R}^n, t_i \in \mathbb{R}^m, i = 1, \ldots, N \} \), an activation function \( g \) and the number of hidden nodes \( M \):

(1) Randomly assign input weights \( w_i \) and biases \( b_j \), \( j = 1, \ldots, M \).

(2) Calculate the hidden layer output matrix \( H \).

(3) Calculate output weights matrix \( \beta = H^T \).

2.2. Localized generalization error model (LGEM)

The localized generalization error model [5,6,10] was proposed by Yeung and it is to find an upper bound of MSE for unseen samples which have similar feature values to the training samples (i.e., the unseen samples will only occur with the distance smaller than a given value \( Q \) from the training samples in the input space) [5,6].

The \( Q \)-neighborhood \( S_Q(x_0) \) of a training sample \( x_0 \) is defined as

\[
S_Q(x_0) = \{ x : |x-x_0| < |x_i-x_0|, i=1,2,\ldots, n \}
\]

where \( n \) denotes the number of input features, and \( Q \) is a given real value. Let \( S_0 \) be the union of all \( S_Q(x_0) \) and it is called as the \( Q \)-union. All samples in \( S_Q(x_0) \) except \( x_0 \) are considered as unseen samples. In classification problem, we usually do not have any knowledge about the distribution of the true input space. Therefore, each unseen sample is expected to have the same chance to appear; correspondingly, \( \Delta x_i \) is considered to be input perturbations which is random variable having uniform distribution with zero mean and variance \( \sigma^2_{\Delta x_i} \). For \( 0 \leq Q_1 < \cdots < Q_k \leq \infty \), the following relationship holds:

\[
D \subseteq S_{Q_1} \subseteq \cdots \subseteq S_{Q_k} \subseteq T
\]

where \( D \) is the training data set and \( T \) is the entire input space. The localized generalization error is defined as:

\[
R_{st}(Q) = \int_{S_Q} (f(x_0)-f(x))^2 p(x) dx
\]

where \( f_0 \) denotes the classifier with parameter set \( \theta \), \( f(x) \) denotes the true unknown input–output mapping function and \( p(x) \)
denotes the true unknown probability density function of the input $x$. By the Hoeffding’s inequality [20], with a probability of $(1 - \eta)$, we have

$$R_{SM}(Q) = \int_{\mathcal{S}}(f_{o}(x) - F_{o}(x))^2p(x)dx \leq (\sqrt{E_{\mathcal{S}}((\Delta y)^2)} + \sqrt{R_{emp} + A})^2 + \varepsilon = R_{SM}^*(Q)$$  \hspace{1cm} (8)

where $\Delta y = f_{o}(x) - f_{0}(x)$, $\varepsilon = B\sqrt{\ln \eta / (-2N)}$, $R_{emp} = \frac{1}{n} \sum_{b=1}^{N}(f_{o}(x_b) - F_{o}(x_b))^2$ - $F_{o}(x_b)$ is the training error, $E_{\mathcal{S}}((\Delta y)^2)$ denotes the stochastic sensitivity measure (SSM), $A$, $B$ and $\eta$, the difference between the maximum and minimum value of the target outputs, the possible maximum value of the MSE and the confidence of the bound, respectively. $A$ and $B$ can be fixed when the data set is given.

3. Architecture selection algorithm based on extended LGEM

The number of hidden neurons in feed-forward neural network is usually selected by sequential learning [9,15,26] or by CV. However, the sequential learning technique determines the number of hidden neurons by making use of the training error without considering the generalization capability. The CV technique is time consuming for large data sets. In this paper, we propose an architecture selection algorithm based on extended LGEM which considers the generalization capability and costs less time than CV. We derive the SSM of $R_{SM}$ for single-hidden layer feed-forward neural network with sigmoid activation function in Section 3.1 and an algorithm for architecture selection of the SLFNs is proposed in Section 3.2.

3.1. Derivation of the SSM for SLFNs with sigmoid activation function

The stochastic sensitivity measure (SSM) measures the output perturbations $|\Delta y|$ of the classifier when the input value changes [21,22,27]. In this paper, we assume the training samples are independent and only consider input perturbation. In addition, the input perturbation of the $i$th input feature is a random variable with a uniform distribution having the mean $\mu_{x_i} = 0$ and the variance $\sigma_{x_i}^2 = (2Q)^2/12 = Q^2/3$. Uniform distribution is assumed here because without any prior knowledge on the distribution of unseen samples, we assume that they have an equal chance of occurrence. According to (1), SLFNs with sigmoid activation function could be described as

$$f_{o}(x) = \sum_{j=1}^{M} \beta_{j} g(w_jx + b_j) = \sum_{j=1}^{M} \beta_{j} \frac{1}{1 + \exp(-(w_jx + b_j))}$$  \hspace{1cm} (9)

According to Taylor’s series expansion: $1/x = \sum_{t=0}^{\infty} (-1)^t x^t$ ($-1 < x < 1$),

$$f_{o}(x) = \sum_{j=1}^{\infty} \beta_{j} \sum_{t=0}^{\infty} (-1)^t (\exp(-(w_jx + b_j)))^t, (0 < \exp(-(w_jx + b_j)) < 1)$$  \hspace{1cm} (10)

Ignoring the terms with order larger than 1, we have:

$$f_{o}(x) \approx \sum_{j=1}^{M} \beta_{j} (1-\exp(-(w_jx + b_j)))$$  \hspace{1cm} (11)

Let $S_j = \sum_{i=0}^{n} (w_{ij}x_i + b_{ij})$ and $S_j^* = \sum_{i=0}^{n} (w_{ij}(x_i + \Delta x_i) + b_{ij})$. Then $E_{SM}((\Delta y)^2)$

$$= E_{\mathcal{S}}(\left(\sum_{j=1}^{M} \beta_{j} (1-\exp(-S_j^*)) - \sum_{j=1}^{M} \beta_{j} (1-\exp(-S_j))\right)^2)$$  \hspace{1cm} (12)

Let $V_j = \exp(-S_j) - \exp(-S_j^*)$, then we have:

$$E_{\mathcal{S}}((\Delta y)^2) \approx \sum_{j=1}^{M} \sum_{j=1}^{M} \beta_{j} \beta_{j'} E_{\mathcal{S}}(V_j^2 | V_j)$$  \hspace{1cm} (13)

We have:

$$E_{\mathcal{S}}(V_j^2 | V_j) = E_{\mathcal{S}}(\exp(-S_j - S_j^*)) - E_{\mathcal{S}}(\exp(-S_j^*)) + E_{\mathcal{S}}(\exp(-S_j))$$  \hspace{1cm} (14)

According to the central limit theorem, $\exp(S_j)$ and $\exp(S_j^*)$ have a log-normal distribution. Therefore,

$$E_{\mathcal{S}}(\exp(-S_j^* - S_j)) = \exp\left(\frac{\text{Var}(S_j^* + S_j)}{2} - E(S_j^* + S_j)\right) \approx 1$$

$$+ \frac{\text{Var}(S_j^* + S_j)}{2} - E(S_j^* + S_j)$$  \hspace{1cm} (15)

Then,

$$E_{\mathcal{S}}(V_j^2 | V_j) = \frac{1}{2} \text{Var}(S_j^* + S_j) + \text{Var}(S_j + S_j) - \text{Var}(S_j + S_j)$$  \hspace{1cm} (16)

Because:

$$\text{Var}(S_j^* + S_j) = \text{Var}\left(\sum_{k=1}^{n} (w_{ik}(x_k + \Delta x_k) + b_{ik})\right) + \sum_{k=1}^{n} (w_{ik}(x_k + \Delta x_k) + b_{ik})^2 = \sum_{k=1}^{n} (w_{ik} + w_{ik})^2 \text{Var}(x_k)$$

And, finally we get:

$$E_{\mathcal{S}}((\Delta y)^2) = \frac{Q^2}{2} \sum_{j=1}^{M} \sum_{j=1}^{M} \beta_{j} \beta_{j'} \sum_{k=1}^{n} w_{ij} w_{ij'}$$  \hspace{1cm} (18)

where $Q$ is a given real value, $\beta_{j}$ is the output of the $j$th hidden node and $w_{ij}$ is the weight connecting the $j$th hidden node and the $k$th input node.

The sensitivity measure (SM) of neural network [21,22] gives a quantified data on the change of network outputs with respect to change of network inputs. Intuitively, it measures how sensitive the network output is to the input change. Here, the formula (18) measures the output fluctuations of the classifier. A classifier that has high output fluctuations yields high SM because its output varies dramatically when the input value changes. Due to the classifier bias/variance dilemma, a classifier yielding a good generalization capability should minimize both the training error and SM or achieves a good balance between the two [28].

3.2. Architecture selection algorithm based on extended LGEM

There are two ways to compare two classifiers [10]. One way is to fix the value of $R_{SM}(Q)$ and compare the magnitude between the $Q$ values. The other is to fix the $Q$ value and compare the $R_{SM}(Q)$. Suppose there are two classifiers, $f_1$ and $f_2$. Given a value $a$ of $R_{SM}$, if $R_{SM}(Q_1) = R_{SM}(Q_2) = a$, $Q_1 < Q_2$, then $f_2$ has a better generalization performance. In order to get the best network structure and find the optimal classifier, we can select the largest $Q$ which satisfies $R_{SM}(Q) = a$. On the other hand, we can also compute the $R_{SM}$ using the same $Q$, the lower $R_{SM}$ is, the better generalization capability the network has.
According to the first way, we can formulate the architecture selection problem as an optimization problem, that is
\[
\max_{Q} R_{SM}^e(Q) \leq a
\]  
(19)

According to (8) and (18), the \( R_{SM}^e \) for SLFNs with sigmoid activation function is as follows:
\[
R_{SM}^e \approx \left( \frac{Q^2}{2} \sum_{j=1}^{M} \sum_{k=1}^{n} w_{jk}^2 + \sqrt{R_{emp} + A} \right)^2 + \epsilon
\]  
(20)

Let \( R_{SM}^e(Q) = a \), we have
\[
Q^2 \sum_{j=1}^{M} \sum_{k=1}^{n} w_{jk}^2 - 3(\sqrt{a-a} - \sqrt{R_{emp} - A})^2 = 0
\]  
(21)

Eq. (21) is a quadratic equation for \( Q \). There are two solutions for \( Q \) and let \( Q^* \) be the positive real solution of the quadratic equation.
\[
h(M, Q^*) = \begin{cases} 
0, & R_{emp} \geq a \\
Q^*, & \text{else}
\end{cases}
\]  
(22)

The algorithm for architecture selection of SLFNs trained with ELM based on LGEM (ASLGEM-ELM) is given as follows.

1) Initialize \( M=1 \);
2) Randomly assign input weight \( w_j \) and bias \( b_j \) of hidden nodes, \( j=1,...,M \);
3) Calculate the hidden layer output matrix \( H \);
4) Calculate output weights matrix \( \beta = H^T \eta \);
5) Calculate the Q-value for the current network using (22);
6) If it does not meet the stop criterion, then \( M=M+1 \) and go to step (2), otherwise, go to (7);
7) Calculate argmax \( h(M, Q^*) \).

The stop criterion could be “\( M \) is equal to the number of training samples”. But it is not only time consuming but also unnecessary. Generally speaking, when \( M \ll N \), the ELM algorithm also gets high training and testing accuracy [11]. In our experiments, we set the stop criterion as \( M=500 \) when the number of training samples is larger than 500.

4. Experimental results and analysis

In this section, we investigate the performance of the proposed algorithm ASLGEM-ELM by conducting experiments on eleven benchmark classification data sets and five regression problems chosen from the University of California at Irvine (UCI) Machine Learning Repository [23]. The basic information of the data sets is given in Table 1. All attributes have been scaled to \([0,1]\) to reduce the effect of large values. Note that the proposed method ASLGEM-ELM can be applied to any variation of problems with any number of features, samples and classes.

In the experiments, every data set is randomly divided into two parts, training data set and testing data set, and the simulations are conducted in Matlab 7.1 running on a desktop PC (2.80 G HZ CPU). Three groups of experiments are conducted for different purposes. In experiment 1, we verify our proposed method with ELM by the results of average testing accuracy. In experiment 2, we compare our method with CV and OP-ELM [31] in architecture selection on classification problems. In experiment 3, it is compared with I-ELM [13], CS-ELM [32] and EM-ELM [15] on regression problems. Each experiment is repeated ten times to get ten independent results for each data set. The experimental results show that our proposed algorithm could obtain a network with the best generalization performance.

![Table 1: The basic information of the benchmarking data sets.](https://example.com/table1)

<table>
<thead>
<tr>
<th>Classification</th>
<th>No. of features</th>
<th>No. of samples</th>
<th>No. of classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iris</td>
<td>4</td>
<td>150</td>
<td>3</td>
</tr>
<tr>
<td>Breast Cancer</td>
<td>10</td>
<td>699</td>
<td>2</td>
</tr>
<tr>
<td>Wine Recognition</td>
<td>13</td>
<td>178</td>
<td>3</td>
</tr>
<tr>
<td>Sonar Target</td>
<td>60</td>
<td>208</td>
<td>2</td>
</tr>
<tr>
<td>Pima Diabetes</td>
<td>8</td>
<td>768</td>
<td>2</td>
</tr>
<tr>
<td>Ionosphere</td>
<td>34</td>
<td>351</td>
<td>2</td>
</tr>
<tr>
<td>Car Evaluation</td>
<td>6</td>
<td>1728</td>
<td>4</td>
</tr>
<tr>
<td>Waveform</td>
<td>21</td>
<td>5000</td>
<td>3</td>
</tr>
<tr>
<td>Optical Digit</td>
<td>64</td>
<td>5620</td>
<td>10</td>
</tr>
<tr>
<td>Mushroom</td>
<td>22</td>
<td>8124</td>
<td>2</td>
</tr>
<tr>
<td>Wisconsin Breast Cancer</td>
<td>32</td>
<td>569</td>
<td>2</td>
</tr>
<tr>
<td>Regression</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Servo</td>
<td>4</td>
<td>167</td>
<td>–</td>
</tr>
<tr>
<td>Boston housing</td>
<td>13</td>
<td>506</td>
<td>–</td>
</tr>
<tr>
<td>Auto-MPG</td>
<td>8</td>
<td>398</td>
<td>–</td>
</tr>
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<td>Ablalone</td>
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<td>4177</td>
<td>–</td>
</tr>
<tr>
<td>Allerones</td>
<td>40</td>
<td>7154</td>
<td>–</td>
</tr>
</tbody>
</table>

The setting of parameters of in ASLGEM-ELM is as follows. In (8), the difference between the maximum and minimum values of target outputs \( A \) and the number of training samples \( N \) are fixed after giving the training data set. The maximum possible value of the MSE \( B \) and the confidence level of the \( R_{emp}^e \) bound \( \eta \) could also be selected before any classifier training. From Eq. (22), one may notice that the smaller a is, the larger number of hidden nodes will be selected by ASLGEM-ELM. This is because the Q value will be zero if its training error is larger than a and the training error decreases as the number of hidden nodes increases. Since the experimental results show that the trained SLFNs with training error larger than 0.25 will not yield good generalization capability, the constant a is set as 0.25.

4.1. Experiment 1: Performance verification

In this section, we verify that the proposed algorithm ASLGEM-ELM with ELM. In Table 2, nodes1 denotes the number of the hidden nodes selected by our algorithm and nodes2 denotes the optimal number of hidden nodes by many trials with ELM. nodes1 \((i=3.4)\) denotes the number of the hidden nodes two times and half as many as nodes2, respectively. From Table 2, the optimal network architecture can be found by our algorithm, because nodes1 and nodes2 is comparative. However, when the number of hidden nodes in SLFNs trained by ELM is more or less than nodes2, the testing accuracies are lower than our method in all the ten datasets. It could be easily observed that our proposed algorithm ASLGEM-ELM could select the optimal network architecture in terms of number of neurons.

4.2. Experiment 2: Benchmarking with real classification applications

In this section, we compare the ASLGEM-ELM with well known architecture selection method, i.e. the k-fold cross validation (CV) [11] and OP-ELM. The average classification testing accuracies and average running time of ten time experiments of each data set are given in Tables 3 and 4.

Experimental results in Table 3 show that the ASLGEM-ELM performs best among the methods in terms of average classification testing accuracy in most cases and training time on all the data sets.
more time is required by CV is that for faster than CV besides waveform and mushroom. The reason that is the fastest in all cases. ASLGEM-ELM is about 5 to 20 times have good generalization capability[24]. The presenting experi-
mental results illustrate that our proposed method has better performance than CV on the same training and testing data sets. As shown in Table 3, for Breast Cancer dataset ASLGEM-ELM gains a little lower testing accuracy than CV, but for all the other data sets, ASLGEM-ELM achieves better generalization performance than others and comparable compact network structure.

From Table 3, the learning speed of our method ASLGEM-ELM is the fastest in all cases. ASLGEM-ELM is about 5 to 20 times faster than CV besides waveform and mushroom. The reason that more time is required by CV is that for k-fold CV and L choices of running times, kL classifiers must be trained. As shown in Table 3, for Wine Recognition dataset ASLGEM-ELM gains a little lower testing accuracy than CV, but for all the other data sets, ASLGEM-ELM gains higher testing accuracies than CV. The reason is that CV method estimates the expected generalization error instead of its bound. Thus, they can not guarantee the constructed classifiers have good generalization capability [24]. The presenting experimental results illustrate that our proposed method has better performance than CV on the same training and testing data sets.

According to the test results of Table 4, the OP-ELM is faster than our method. This is comprehensible, because our method trains multiple classifiers in order to find the maximum value of Q (i.e., the classifier with the best generalization performance). It could be obviously concluded that the network architecture selected by our method is superior than that of the OP-ELM selected.

### 4.3. Experiment 3: Benchmarking with regression applications

In this section, we evaluate and compare the performance of the proposed ASLGEM-ELM with I-ELM, EM-ELM and CS-ELM. In our experiments, the I-ELM and EM-ELM algorithms have two parameters that need to be considered: the expected training rate e and the maximum number of hidden nodes \( L_{\text{max}} \). In the evaluations, we aim to get the network when the training rate reaches the specific e without the constraint of \( L_{\text{max}} \). Therefore, in our experiments, \( L_{\text{max}} \) is set to be a large value of 2000. For the CS-ELM, only the number of hidden neurons needs to be set by us. According to [32], \( L_{\text{max}} \) is set to be 100 in order to obtain a compact network structure.

From Table 5, it is clear that the proposed ASLGEM-ELM achieves better generalization performance than others and comparable compact network structure.

I-ELM achieves comparable performance with the largest network structure for all the regression cases. Comparing CS-ELM, EM-ELM with the I-ELM, the network obtained using CS-ELM and EM-ELM is always more compact than I-ELM. From Table 5, ASLGEM-ELM performs better than others, because our method is proposed based on generalization error.

### 5. Conclusions and future works

In this paper, we derive the SSM of SLFNs with sigmoid activation function and propose an algorithm for architecture selection of SLFNs trained by ELM based on LGEM. The algorithm is not only simple but also efficient to automatically determine the number of hidden nodes with the best generalization performance. The experimental results demonstrate that the proposed method could select the architecture of SLFNs which have the best generalization performance, and yield higher average classification testing accuracy and less training time than CV.

Since ELM randomly generates input weights and biases which may differ each time, the network structure is not steady. In our experiments, we have to compute the average number of hidden nodes. In this paper, we have considered the input
perturbation, leaving behind the weights perturbation. A future work is to add the effect of weights perturbation to the SSM.

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