A New Type-2 Intuitionistic Exponential Triangular Fuzzy Number and Its Ranking Method with Centroid Concept and Euclidean Distance

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Abstract*—This paper introduces a new type-2 intuitionistic exponential triangular fuzzy number. Basic generalized exponential triangular intuitionistic fuzzy numbers are formulated by (α, β)-cuts. Some of properties and theorems of this type of fuzzy number with graphical representations have been studied and some examples are given to show the effectiveness of the proposed method. Also, the ranking function of the generalized exponential triangular intuitionistic fuzzy number is computed. This ranking method is based on the centroid concept and Euclidean distance. Based on the ranking method, we develop an approach to solving an intuitionistic fuzzy assignment problem where cost is not deterministic numbers but imprecise ones. Then, we solve an intuitionistic fuzzy transportation problem where transportation cost, source, and demand were generalized type-2 intuitionistic fuzzy numbers by the ranking method for Euclidean distance. Intuitionistic fuzzy set theory has been used for analyzing the fuzzy system reliability. We have taken the intuitionistic fuzzy failure to start of an automobile as known basic fault events such as Ignition failure, Battery internal shortage, Spark plug failure and fuel pump failure using Type-2 Intuitionistic Exponential Triangular Fuzzy Number. Our computational procedure is very simple to implement for calculations in intuitionistic fuzzy failure. The major advantage of using Intuitionistic fuzzy sets over fuzzy sets is that intuitionistic fuzzy sets separate the positive and the negative evidence for the membership of an element in a set. Furthermore, the proposed technique can be suitably utilized to solve the start of an automobile problem, because the result of system failure in this method is significant. Finally, the proposed method has been compared with other existing method through numerical examples.

I. Introduction

Bellman and Zadeh [1] initially proposed the basic model of fuzzy decision making based on the theory of fuzzy mathematics. Centroid concept in ranking fuzzy number only started in 1980 by Yager [12]. Yager was the first researcher who contributed the centroid concept in the ranking method and used the horizontal coordinate x as the ranking index. Cheng [13] argued in certain cases, the value of x can also be an aid index and y becomes the important index especially when the values of x are equal or the left and right spread are the same for all fuzzy numbers. Also, Rezvani [14] proposed ranking generalized trapezoidal fuzzy numbers with Euclidean distance by the incentre of centroids. The intuitionistic fuzzy set (IFS) is an extension of fuzzy set. IFS was first introduced by Atanassov[2]. Fuzzy sets are characterized by the membership function only, but IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one [3]. For the comparison of fuzzy numbers, there are many different methods [4]-[8]. The ranking of fuzzy numbers is important in fuzzy multi attribute decision making (MADM). Recently, the IFN receives little attention and different definitions of IFNs have been proposed as well as the corresponding ranking methods of IFNs. Rezvani[9] proposed ranking approach based on values and ambiguities of the membership degree and the non-membership degree for trapezoidal intuitionistic fuzzy number. Jana[11] developed a new deffinition on type-2 intuitionistic fuzzy and transportation problem.

The concept of fault tree analysis (FTA) was developed in 1962 at Bell Telephone Laboratories. FTA is a logical and diagrammatic method for evaluating system reliability. It is a logical approach for systematically quantifying the possibility of abnormal system event. For such systems, it is, therefore, unrealistic to assume a crisp number (classical) for different basic events. Suresh et al. [15] used a method based on α-cuts to deal with FTA, treating the failure possibility as triangular and trapezoidal fuzzy numbers. G. S. Mahapatra et al. [16] proposed fuzzy fault tree analysis using intuitionistic fuzzy numbers. Tyagi et al. [17] used fuzzy set to analysis fuzzy fault tree. Mahapatra and Roy [18] presented triangular intuitionistic fuzzy number and used it for reliability evaluation. Singer [19] proposed a method using fuzzy numbers to represent the relative frequencies of the basic events. He used possibilistic AND and OR operators to construct possible fault tree. Wang J. Q. et al. [20] provided new operators on triangular intuitionistic fuzzy numbers and their applications in system fault analysis. Neeraj Lata [21] presented the fuzzy fault tree analysis using intuitionistic fuzzy numbers.

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II. Preliminaries

Definition 1. (Intuitionistic Triangular Fuzzy Number) A $ITFN$ $\tilde{a} = (\langle \underline{a}, a, \bar{a} \rangle; \mu_{\tilde{a}}, \nu_{\tilde{a}})$ is a set on a real number set $\mathcal{R}$, whose membership function and non-membership function are defined as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} w_{\tilde{a}} \ \frac{x-\underline{a}}{a-\underline{a}} & \underline{a} \leq x \leq a, \\ w_{\tilde{a}} & x = a, \\ w_{\tilde{a}} \ \frac{\bar{a}-x}{\bar{a}-a} & a \leq x \leq \bar{a}, \\ 0 & Otherwise. \end{cases} \quad (1)$$

$$\nu_{\tilde{a}}(x) = \begin{cases} w_{\tilde{a}} \ \frac{a-x}{a-\underline{a}'} & \underline{a}' \leq x \leq a, \\ w_{\tilde{a}} & x = a, \\ w_{\tilde{a}} \ \frac{x-a}{\bar{a}'-a} & a \leq x \leq \bar{a}', \\ 1 & Otherwise. \end{cases} \quad (2)$$

A $ITFN$ $\tilde{a}$ is called $GITFN$, if the following hold:
(i) There exists $x \in \mathcal{R}$, $\mu_{\tilde{a}}(x) = w,\ \nu_{\tilde{a}}(x) = 0,\ 0 < w \leq 1$.
(ii) $\mu_{\tilde{a}}$ is continuous mapping from $\mathcal{R}$ to the closed interval $[0, w]$ and $x \in \mathcal{R}$, the relation $0 \leq \mu_{\tilde{a}}(x) + \nu_{\tilde{a}}(x) \leq w$ holds.

Definition 2. (Interval-valued Triangular Fuzzy Number) An interval-valued fuzzy set $\tilde{a}$ on $\mathcal{R}$ is given by $\tilde{a} = \{(x, [\mu_{\tilde{a}^L}(x), \mu_{\tilde{a}^U}(x)])\}$, $\forall x \in \mathcal{R}$. where $o \leq \mu_{\tilde{a}^L}(x) \leq \mu_{\tilde{a}^U}(x) \leq 1$ and $\mu_{\tilde{a}^L}(x), \mu_{\tilde{a}^U}(x) \in [0, 1]$ and denoted by $\mu_{\tilde{a}}(x) = [\mu_{\tilde{a}^L}(x), \mu_{\tilde{a}^U}(x)]$ or $\tilde{a} = [\tilde{a}^L, \tilde{a}^U]$, $x \in \mathcal{R}$.
The interval-valued fuzzy set $\tilde{a}$ indicates that, when the membership grade of X belongs to the interval $[\mu_{\tilde{a}^L}(x), \mu_{\tilde{a}^U}(x)]$, the largest grade is $\mu_{\tilde{a}^U}(x)$ and the smallest grade is $\mu_{\tilde{a}^L}(x)$. Let

$$\mu_{\tilde{a}}(x) = \begin{cases} w_{\tilde{a}} \ \frac{x-\underline{a}}{a-\underline{a}} & \underline{a} \leq x \leq a, \\ w_{\tilde{a}} & x = a, \\ w_{\tilde{a}} \ \frac{\bar{a}-x}{\bar{a}-a} & a \leq x \leq \bar{a}, \\ 0 & Otherwise. \end{cases} \quad (3)$$

$$\nu_{\tilde{a}}(x) = \begin{cases} w_{\tilde{a}} \ \frac{a-x}{a-\underline{a}'} & \underline{a}' \leq x \leq a, \\ w_{\tilde{a}} & x = a, \\ w_{\tilde{a}} \ \frac{x-a}{\bar{a}'-a} & a \leq x \leq \bar{a}', \\ 1 & Otherwise. \end{cases} \quad (4)$$

Then $\tilde{a}^L = (\underline{a}, a, \bar{a}; \lambda)$, $\underline{a} < a < \bar{a}$ and $\tilde{a}^U = (\underline{a}', a, \bar{a}'; \rho)$, $\underline{a}' < a < \bar{a}'$.
Consider the case in which $0 < \lambda \leq \rho \leq 1$ and $\underline{a}' < \underline{a} < a < \bar{a} < \bar{a}'$. So $\tilde{a} = [\tilde{a}^L, \tilde{a}^U] = [(\underline{a}, a, \bar{a}; \lambda), (\underline{a}', a, \bar{a}'; \rho)]$.

III. A New Type-2 Intuitionistic Exponential Triangular Fuzzy Number

Definition 3. (Interval-valued Exponential Triangular Fuzzy Number) Let $\tilde{a}$ be exponential fuzzy number $\tilde{a} = (a, b, c; w)_E$. Membership function of an interval-valued exponential triangular fuzzy number is defined as follows:

$$\mu_{\tilde{a}^L}(x) = \begin{cases} \lambda \ e^{-(\frac{b_1-x}{b_1-a_1})} & a_1 \leq x \leq b_1, \\ \lambda \ e^{-(\frac{x-b_1}{c_1-b_1})} & b_1 \leq x \leq c_1, \\ 0 & Otherwise. \end{cases} \quad (5)$$

$$\mu_{\tilde{a}^U}(x) = \begin{cases} \lambda \ e^{-(\frac{b_2-x}{b_2-a_2})} & a_2 \leq x \leq b_2, \\ \lambda \ e^{-(\frac{x-b_2}{c_2-b_2})} & b_2 \leq x \leq c_2, \\ 0 & Otherwise. \end{cases} \quad (6)$$

Definition 4. (Intuitionistic Exponential Triangular Fuzzy Number) A $IETFN$ $\tilde{a} = (\langle \underline{a}, a, \bar{a} \rangle; \mu_{\tilde{a}}, \nu_{\tilde{a}})_E$ with membership and non-membership functions is defined as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} w \ e^{-(\frac{a-x}{a-\underline{a}})} & \underline{a} \leq x \leq a, \\ w & w = a, \\ w \ e^{-(\frac{x-a}{\bar{a}-a})} & a \leq x \leq \bar{a}, \\ 0 & Otherwise. \end{cases} \quad (7)$$

$$\nu_{\tilde{a}}(x) = \begin{cases} w \ e^{-(\frac{x-\underline{a}'}{a-\underline{a}'})} & \underline{a}' \leq x \leq a, \\ w & w = a, \\ w \ e^{-(\frac{\bar{a}'-x}{\bar{a}'-a})} & a \leq x \leq \bar{a}', \\ 0 & Otherwise. \end{cases} \quad (8)$$

Definition 5. (Type-2 Intuitionistic Exponential Triangular Fuzzy Number) Let $\tilde{a}$ be a $T2IETFN$ $\tilde{a}^* = [\mu_{\tilde{a}^L}(x), \mu_{\tilde{a}^U}(x), \nu_{\tilde{a}^L}(x), \nu_{\tilde{a}^U}(x)]$ in $\mathcal{R}$ and defined as $(a_1^*, b_1^*, c_1^*; w^*)(a_2^*, b_2^*, c_2^*; w^*)$ with $a_2 \leq a_1 \leq b_2 \leq b_1 \leq c_1 \leq c_2$ has membership and non-membership function as follows:

$$\mu_{\tilde{a}^*}(x) = \begin{cases} w^* \ e^{-(\frac{b^*-x}{b^*-a^*})} & a^* \leq x \leq b^*, \\ w^* \ e^{-(\frac{x-b^*}{c^*-b^*})} & b^* \leq x \leq c^*, \\ 0 & Otherwise. \end{cases} \quad (9)$$

$$\nu_{\tilde{a}^*}(x) = \begin{cases} w^* \ e^{-(\frac{x-a'^*}{b^*-a'^*})} & a'^* \leq x \leq b^*, \\ w^* \ e^{-(\frac{c'^*-x}{c'^*-b^*})} & b^* \leq x \leq c'^*, \\ 0 & Otherwise. \end{cases} \quad (10)$$

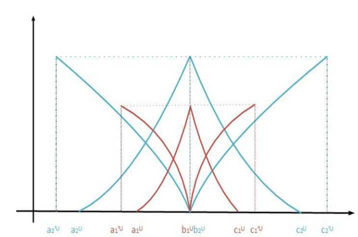

Fig. 1. Type-2 intuitionistic exponential triangular fuzzy number

IV. The basic concept of the $T2IETFN$

Definition 6. The α-cut set of $\tilde{a}^* = (a_1^*, b_1^*, c_1^*; w^*)(a_2^*, b_2^*, c_2^*; w^*)$ is a crisp subset of $\mathcal{R}$, which is defined as $\tilde{a}^*_\alpha = \{x : \mu_{\tilde{a}^*}(x) \geq \alpha\}$. Using $\mu_{\tilde{a}^*}(x)$ and definition of α-cut is found that $\tilde{a}^*_\alpha$ is a closed interval, denoted by:

$$(1)\ w^* \ e^{-(\frac{b^*-x}{b^*-a^*})} = \alpha \Rightarrow \frac{x-b^*}{b^*-a^*} = \ln \frac{\alpha}{w^*}$$

$$\Rightarrow x = b^* + (\ln \frac{\alpha}{w^*})(b^* - a^*)$$

$$(2)\ w^*\ e^{-(\frac{x-b^*}{c^*-b^*})} = \alpha \Rightarrow \frac{b^*-x}{c^*-b^*} = \ln\frac{\alpha}{w^*}$$

$$\Rightarrow x = b^* - (\ln\frac{\alpha}{w^*})(c^*-b^*)$$

$$(1),(2) \Longrightarrow \tilde{a}^*_\alpha = [L_\alpha(\tilde{a}^*), R_\alpha(\tilde{a}^*)]$$

$$= [b^* + (\ln\frac{\alpha}{w^*})(b^*-a^*), b^* - (\ln\frac{\alpha}{w^*})(c^*-b^*)] \quad (11)$$

Definition 7. The β-cut set of $\tilde{a}^* = (a_1^*, b_1^*, c_1^*; w^*)(a_2^*, b_2^*, c_2^*; w^*)$ is a crisp subset of $\mathcal{R}$, which is defined as $\tilde{a}^*_\beta = \{x : \mu_{\tilde{a}^*}(x) \le \beta\}$. Using $\mu_{\tilde{a}^*}(x)$ and definition of β-cut is found that $\tilde{a}^*_\alpha$ is a closed interval, denoted by:

$$(1)\ w^*\ e^{-(\frac{x-a'^*}{b^*-a'^*})} = \beta \Rightarrow \frac{a'^*-x}{b^*-a'^*} = \ln\frac{\beta}{w^*}$$

$$\Rightarrow x = a'^* - (\ln\frac{\beta}{w^*})(b^*-a'^*)$$

$$(2)\ w^*\ e^{-(\frac{c'^*-x}{c'^*-b^*})} = \beta \Rightarrow \frac{x-c'^*}{c'^*-b^*} = \ln\frac{\beta}{w^*}$$

$$\Rightarrow x = c'^* + (\ln\frac{\beta}{w^*})(c'^*-b^*)$$

$$(1),(2) \Longrightarrow \tilde{a}^*_\beta = [L_\beta(\tilde{a}^*), R_\beta(\tilde{a}^*)]$$

$$= [a'^* - (\ln\frac{\beta}{w^*})(b^*-a'^*), c'^* + (\ln\frac{\beta}{w^*})(c'^*-b^*)] \quad (12)$$

Definition 8. The (α,β)-cut $T2IETFN$ is defined by $\tilde{a}^*_{\alpha,\beta} = \{[L_\alpha(\tilde{a}^*), R_\alpha(\tilde{a}^*)], [L_\beta(\tilde{a}^*), R_\beta(\tilde{a}^*)]\},\ \alpha+\beta \le w, \quad \alpha,\beta \in [0,1]$. Where

$$\begin{cases} L_\alpha(\tilde{a}^*) = b^* + (\ln\frac{\alpha}{w^*})(b^*-a^*) \\ R_\alpha(\tilde{a}^*) = b^* - (\ln\frac{\alpha}{w^*})(c^*-b^*) \\ L_\beta(\tilde{a}^*) = a'^* - (\ln\frac{\beta}{w^*})(b^*-a'^*) \\ R_\beta(\tilde{a}^*) = c'^* + (\ln\frac{\beta}{w^*})(c'^*-b^*). \end{cases} \quad (13)$$

Theorem 1. If $\tilde{a}^* = (a_1^*, b_1^*, c_1^*; w_1^*)(a_2^*, b_2^*, c_2^*; w_1^*)$ and $\tilde{z}^* = (a_3^*, b_3^*, c_3^*; w_2^*)(a_4^*, b_4^*, c_4^*; w_2^*)$ are two $T2IETFN$, then $\tilde{a}^* \oplus \tilde{z}^* = (a_1^*+a_3^*, b_1^*+b_3^*, c_1^*+c_3^*; w^*)(a_2^*+a_4^*, b_2^*+b_4^*, c_2^*+c_4^*; w^*)$ is also a $T2IETFN$, where $0 < w \le 1,\ w^* = min(w_1^*, w_2^*)$.

Theorem 2. If $\tilde{a}^* = (a_1^*, b_1^*, c_1^*; w_1^*)(a_2^*, b_2^*, c_2^*; w_1^*)$ and $\tilde{z}^* = (a_3^*, b_3^*, c_3^*; w_2^*)(a_4^*, b_4^*, c_4^*; w_2^*)$ are two $T2IETFN$, then $\tilde{a}^* \ominus \tilde{z}^* = (a_1^*-c_3^*, b_1^*-b_3^*, c_1^*-a_3^*; w^*)(a_2^*-c_4^*, b_2^*-b_4^*, c_2^*-a_4^*; w^*)$ is also a $T2IETFN$, where $0 < w \le 1,\ w^* = min(w_1^*, w_2^*)$.

Theorem 3. If $\tilde{a}^* = (a_1^*, b_1^*, c_1^*; w^*)(a_2^*, b_2^*, c_2^*; w^*)$ be a $T2IETFN$, then $C\tilde{a}^*$ is $T2IETFN$ and if $C > 0$ then, $C\tilde{a}^* = (Ca_1^*, Cb_1^*, Cc_1^*; w^*)(Ca_2^*, Cb_2^*, Cc_2^*; w^*)$ and $C < 0$ then, $C\tilde{a}^* = (Cc_1^*, Cb_1^*, Ca_1^*; w^*)(Cc_2^*, Cb_2^*, Ca_2^*; w^*)$.

Theorem 4. If $\tilde{a}^* = (a_1^*, b_1^*, c_1^*; w_1^*)(a_2^*, b_2^*, c_2^*; w_1^*)$ and $\tilde{z}^* = (a_3^*, b_3^*, c_3^*; w_2^*)(a_4^*, b_4^*, c_4^*; w_2^*)$ are two $T2IETFN$, then the division of two $T2IETFN$, $\frac{\tilde{a}^*}{\tilde{z}^*} = (\frac{a_1^*}{c_3^*}, \frac{b_1^*}{b_3^*}, \frac{c_1^*}{a_3^*}; w^*)(\frac{a_2^*}{c_4^*}, \frac{b_2^*}{b_4^*}, \frac{c_2^*}{a_4^*}; w^*)$ is also a $T2IETFN$, where $0 < w \le 1,\ w^* = min(w_1^*, w_2^*)$.

Theorem 5. If $\tilde{a}^* = (a_1^*, b_1^*, c_1^*; w_1^*)(a_2^*, b_2^*, c_2^*; w_1^*)$ and $\tilde{z}^* = (a_3^*, b_3^*, c_3^*; w_2^*)(a_4^*, b_4^*, c_4^*; w_2^*)$ are two $T2IETFN$, then the multiplication of two $T2IETFN$, $\tilde{a}^* \otimes \tilde{z}^* = (a_1^*a_3^*, b_1^*b_3^*, c_1^*c_3^*; w^*)(a_2^*a_4^*, b_2^*b_4^*, c_2^*c_4^*; w^*)$ is also a $T2IETFN$, where $0 < w \le 1,\ w^* = min(w_1^*, w_2^*)$.

V. Example of Basic Concept

Example 1. If $\tilde{a}^* = [(2,5,8;0.5)(1,5,9;0.5),(0,5,10;0.7)(-1,5,11;0.7))]$ and $\tilde{z}^* = [(-1,2,5;0.5)(-2,2,6;0.5),(-3,2,7;0.7)(-4,2,8;0.7))]$ be two $T2IETFN$, then the addition of those two fuzzy number is:

$$\mu^L_{\tilde{a}^*}(x) = \begin{cases} 0.5e^{-(\frac{5-x}{5-2})} & 2 \le x \le 5, \\ 0.5 & x = 5, \\ 0.5e^{-(\frac{x-5}{8-5})} & 5 \le x \le 8, \\ 0 & Otherwise. \end{cases}$$

$$\mu^U_{\tilde{a}^*}(x) = \begin{cases} 0.7e^{-(\frac{5-x}{5-0})} & 0 \le x \le 5, \\ 0.7 & x = 5, \\ 0.7e^{-(\frac{x-5}{10-5})} & 5 \le x \le 10, \\ 0 & Otherwise. \end{cases}$$

$$\nu^L_{\tilde{a}^*}(x) = \begin{cases} 0.5e^{-(\frac{x-1}{5-1})} & 1 \le x \le 5, \\ 0.5 & x = 5, \\ 0.5e^{-(\frac{9-x}{9-5})} & 5 \le x \le 9, \\ 0 & Otherwise. \end{cases}$$

$$\nu^U_{\tilde{a}^*}(x) = \begin{cases} 0.7e^{-(\frac{x+1}{5+1})} & -1 \le x \le 5, \\ 0.7 & x = 5, \\ 0.7e^{-(\frac{11-x}{11-5})} & 5 \le x \le 11, \\ 0 & Otherwise. \end{cases}$$

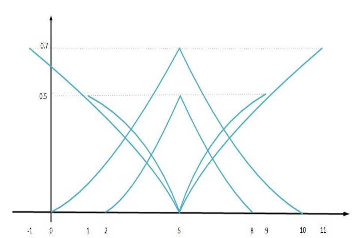

Fig. 2. $T2IETFN$ of $\tilde{a}^*$

$$\mu^L_{\tilde{z}^*}(x) = \begin{cases} 0.5e^{-(\frac{2-x}{2+1})} & -1 \le x \le 2, \\ 0.5 & x = 2, \\ 0.5e^{-(\frac{x-2}{5-2})} & 2 \le x \le 5, \\ 0 & Otherwise. \end{cases}$$

$$\mu^U_{\tilde{z}^*}(x) = \begin{cases} 0.7e^{-(\frac{2-x}{2+3})} & -3 \le x \le 2, \\ 0.7 & x = 2, \\ 0.7e^{-(\frac{x-2}{7-2})} & 2 \le x \le 7, \\ 0 & Otherwise. \end{cases}$$

$$\nu^L_{\tilde{z}^*}(x) = \begin{cases} 0.5e^{-(\frac{x+2}{2+2})} & -2 \le x \le 2, \\ 0.5 & x = 2, \\ 0.5e^{-(\frac{6-x}{6-2})} & 2 \le x \le 6, \\ 0 & Otherwise. \end{cases}$$

$$\nu^U_{\tilde{z}^*}(x) = \begin{cases} 0.7e^{-(\frac{x+4}{2+4})} & -4 \le x \le 2, \\ 0.7 & x = 2, \\ 0.7e^{-(\frac{8-x}{8-2})} & 2 \le x \le 8, \\ 0 & Otherwise. \end{cases}$$

So addition of those two fuzzy number is

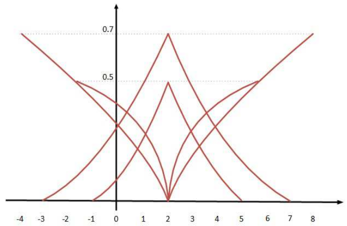

Fig. 3. $T2IETFN$ of $\tilde{z}^*$

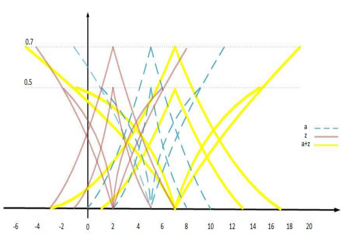

Fig. 4. Addition of $\tilde{a}^* \oplus \tilde{z}^*$

Example 2. If $\tilde{a}^* = [(2,5,8;0.5)(1,5,9;0.5)$ $,(0,5,10;0.7)(-1,5,11;0.7))]$ and $\tilde{z}^* = [(-1,2,5;0.5)(-2,2,6;0.5),(-3,2,7;0.7)(-4,2,8;0.7))]$ be two $T2IETFN$, then the substraction of those two fuzzy number is:

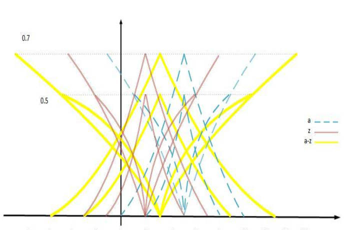

Fig. 5. Substraction of $\tilde{a}^* \ominus \tilde{z}^*$

Example 3. If $\tilde{a}^* = [(-1,2,5;0.5)(-2,2,6;0.5),$ $(-3,2,7;0.7)(-4,2,8;0.7))]$ be a $T2IETFN$ and C is constant, then the multiplication of this fuzzy number with constant number is:
First $C = 2$, Second $C = -2$,

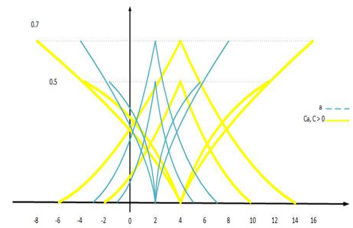

Fig. 6. $C\tilde{a}^*$, $C > 0$ for $T2IETFN$

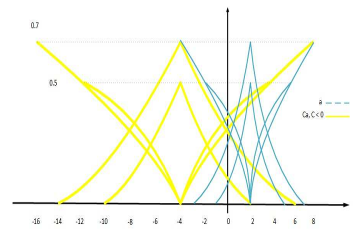

Fig. 7. $C\tilde{a}^*$, $C < 0$ for $T2IETFN$

Example 4. If $\tilde{a}^* = [(4,5,8;0.5)(3,5,9;0.5),$ $(2,5,10;0.7)(1,5,11;0.7))]$ and $\tilde{z}^* = [(-1,2,5;0.5)(-2,2,6;0.5),(-3,2,7;0.7)(-4,2,8;0.7))]$ be two $T2IETFN$, then the division$(\frac{\tilde{z}^*}{\tilde{a}^*})$ of those two fuzzy number is:

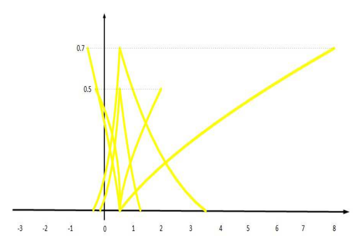

Fig. 8. division$(\frac{\tilde{z}^*}{\tilde{a}^*})$ for $T2IETFN$

VI. Ranking Fuzzy numbers by Centroid Concept

Definition 9. Let $\tilde{a}$ be a $T2IETFN$ $\tilde{a}^* = [\mu_{\tilde{a}^L}(x), \mu_{\tilde{a}^U}(x), \nu_{\tilde{a}^L}(x), \nu_{\tilde{a}^U}(x)]$ in $\mathcal{R}$ defined as:

$$\mu_{\tilde{a}^*}(x) = \begin{cases} f_{\tilde{a}^*}(x) & a^* \le x \le b^*, \\ g_{\tilde{a}^*}(x) & b^* \le x \le c^*, \\ 0 & Otherwise. \end{cases} \tag{14}$$

$$\nu_{\tilde{a}^*}(x) = \begin{cases} f'_{\tilde{a}^*}(x) & a'^* \le x \le b^*, \\ g'_{\tilde{a}^*}(x) & b^* \le x \le c'^*, \\ 0 & Otherwise. \end{cases} \tag{15}$$

where $0 \le \mu_{\tilde{a}^*}(x) + \nu_{\tilde{a}^*}(x) \le 1$ and $a'_2 \le a_2 \le a'_1 \le a_1 \le b_1 \le c_1 \le c'_1 \le c_2 \le c'_2$, and functions $f_{\tilde{a}^*}(x), g_{\tilde{a}^*}(x), f'_{\tilde{a}^*}(x), g'_{\tilde{a}^*}(x) : \mathcal{R} \to [0,1]$ are called the legs of membership function $\mu_{\tilde{a}^*}(x)$ and non-membership function $\nu_{\tilde{a}^*}(x)$. The function $f_{\tilde{a}^*}(x), g'_{\tilde{a}^*}(x)$ and $g_{\tilde{a}^*}(x), f'_{\tilde{a}^*}(x)$ are non-decreasing continuous functions and the functions $g_{\tilde{a}^*}(x), f'_{\tilde{a}^*}(x)$ are non-increasing continuous functions. Therefore inverse functions can be defined as follows:

$$\begin{cases} L_f^{-1}(\tilde{a}^*) = b^* + (\ln \frac{y}{w^*})(b^* - a^*) \\ R_f^{-1}(\tilde{a}^*) = b^* - (\ln \frac{y}{w^*})(c^* - b^*) \\ L_g^{-1}(\tilde{a}^*) = a'^* - (\ln \frac{y}{w^*})(b^* - a'^*) \\ R_g^{-1}(\tilde{a}^*) = c'^* + (\ln \frac{y}{w^*})(c'^* - b^*). \end{cases}$$

Yager [14] was the first researcher to proposed a centroid-index ranking method to calculate the value x_0 for a fuzzy number A as

$$x_0 = \frac{\int_0^1 w(x)A(x)dx}{\int_0^1 A(x)dx}$$

where $w(x)$ is a weighting function measuring the importance of the value x and $A(x)$ denotes the membership function of the fuzzy number A. The larger the value is of x_0 the better ranking of A.

The method of ranking trapezoidal intuitionistic fuzzy numbers with centroid index uses the geometric center of a trapezoidal intuitionistic fuzzy number. The geometric center corresponds to x_A value on the horizontal axis and y_A value on the vertical axis.

Cheng[13] used a centroid-based distance approach to rank fuzzy numbers. For trapezoidal fuzzy number $A = (a, b, c, d; w)$, the distance index can be defined as

$$R(A) = \sqrt{x_0^2 + y_0^2}$$

$$Where\ x_0 = \frac{\int_a^b xL_A(x)dx + \int_b^c xdx + \int_c^d xU_A(x)dx}{\int_a^b L_A(x)dx + \int_b^c dx + \int_c^d U_A(x)dx},$$

$$and\ y_0 = w\frac{\int_0^1 yA_L(y)dy + \int_0^1 yA_U(y)dy}{\int_0^1 A_L(y)dy + \int_0^1 A_U(y)dy}.$$

U_A and L_A are the respective right and left membership function of A, and A_U and A_L, are the inverse of U_A and L_A respectively.

Definition 10. The centroid point $(x_{\tilde{a}^*}, y_{\tilde{a}^*})$ of the $T2IETFN$ $\tilde{a}^*$ is determined as follows:

$$x_\mu(\tilde{a}^*) = \frac{\int xf_{\tilde{a}^*}^L(x)dx + \int yf_{\tilde{a}^*}^R(x)dx}{\int f_{\tilde{a}^*}^L(x)dx + \int f_{\tilde{a}^*}^R(x)dx}$$

$$x_\mu(\tilde{a}^*) = \tag{16}$$

$$\frac{\int_{a_1}^{b_1} xL_{f_1}(\tilde{a}^*)dx + \int_{a_2}^{b_1} xL_{f_2}(\tilde{a}^*)dx + \int_{b_1}^{c_1} xR_{f_1}(\tilde{a}^*)dx + \int_{b_1}^{c_2} xR_{f_2}(\tilde{a}^*)dx}{\int_{a_1}^{b_1} L_{f_1}(\tilde{a}^*)dx + \int_{a_2}^{b_1} L_{f_2}(\tilde{a}^*)dx + \int_{b_1}^{c_1} R_{f_1}(\tilde{a}^*)dx + \int_{b_1}^{c_2} R_{f_2}(\tilde{a}^*)dx}$$

$$x_\nu(\tilde{a}^*) = \frac{\int xg_{\tilde{a}^*}^L(x)dx + \int xg_{\tilde{a}^*}^R(x)dx}{\int g_{\tilde{a}^*}^L(x)dx + \int g_{\tilde{a}^*}^R(x)dx}$$

$$x_\nu(\tilde{a}^*) = \tag{17}$$

$$\frac{\int_{a_1'}^{b_1} xL_{g_1}(\tilde{a}^*)dx + \int_{a_2'}^{b_1} xL_{g_2}(\tilde{a}^*)dx + \int_{b_1}^{c_1'} xR_{g_1}(\tilde{a}^*) + \int_{b_1}^{c_2'} xR_{g_2}(\tilde{a}^*)dx}{\int_{a_1'}^{b_1} L_{g_1}(\tilde{a}^*)dx + \int_{a_2'}^{b_1} L_{g_2}(\tilde{a}^*)dx + \int_{b_1}^{c_1'} R_{g_1}(\tilde{a}^*) + \int_{b_1}^{c_2'} R_{g_2}(\tilde{a}^*)dx}$$

and

$$y_\mu(\tilde{a}^*) = \frac{\int_0^1 yf_{\tilde{a}^*}^{L^{-1}}(y)dy - \int_0^1 yf_{\tilde{a}^*}^{R^{-1}}(y)dy}{\int_0^1 f_{\tilde{a}^*}^{L^{-1}}(y)dy - \int_0^1 f_{\tilde{a}^*}^{R^{-1}}(y)dy}$$

$$y_\mu(\tilde{a}^*) = \frac{\int_0^1 y(L_{f_1}^{-1}(\tilde{a}^*) + L_{f_2}^{-1}(\tilde{a}^*))dy - \int_0^1 y(R_{f_1}^{-1}(\tilde{a}^*) + R_{f_2}^{-1}(\tilde{a}^*))dy}{\int_0^1 (L_{f_1}^{-1}(\tilde{a}^*) + L_{f_2}^{-1}(\tilde{a}^*))dy - \int_0^1 (R_{f_1}^{-1}(\tilde{a}^*) + R_{f_2}^{-1}(\tilde{a}^*))dy} \tag{18}$$

$$y_\nu(\tilde{a}^*) = \frac{\int_0^1 yg_{\tilde{a}^*}^{L^{-1}}(y)dy - \int_0^1 yg_{\tilde{a}^*}^{R^{-1}}(y)dy}{\int_0^1 g_{\tilde{a}^*}^{L^{-1}}(y)dy - \int_0^1 g_{\tilde{a}^*}^{R^{-1}}(y)dy}$$

$$y_\nu(\tilde{a}^*) = \frac{\int_0^1 y(L_{g_1}^{-1}(\tilde{a}^*) + L_{g_2}^{-1}(\tilde{a}^*))dy - \int_0^1 y(R_{g_1}^{-1}(\tilde{a}^*) + R_{g_2}^{-1}(\tilde{a}^*))dy}{\int_0^1 (L_{g_1}^{-1}(\tilde{a}^*) + L_{g_2}^{-1}(\tilde{a}^*))dy - \int_0^1 (R_{g_1}^{-1}(\tilde{a}^*) + R_{g_2}^{-1}(\tilde{a}^*))dy} \tag{19}$$

Theorem 6. If $\tilde{a}^* = (a_1^*, b_1^*, c_1^*; w_1^*)(a_2^*, b_2^*, c_2^*; w_1^*)$ is a $T2IETFN$, then centroid point of $\tilde{a}^*$ is

$$(i)\ x_\mu(\tilde{a}^*) = \tag{20}$$

$$\frac{(e-3)(b_1a_1 + b_1a_2 - b_1c_1 - b_1c_2) + (2-e)(a_1^2 + a_2^2 - c_1^2 - c_2^2)}{(e-1)(c_1 + c_2 - a_1 - a_2)}$$

$$(ii)\ x_\nu(\tilde{a}^*) = \frac{(e-3)(b_1c_1' - b_1c_2' - b_1a_1' - b_1a_2')}{(e-1)(c_1' + c_2' - a_1' - a_2')}, \tag{21}$$

and

$$(iii)\ y_\mu(\tilde{a}^*) = \frac{(\frac{c_1+c_2-a_1-a_2}{2})\ln\frac{1}{w} + \frac{a_1+a_2-c_1-c_2}{4}}{(c_1+c_2-a_1-a_2)\ln\frac{1}{w} + (a_1+a_2-c_1-c_2)}, \tag{22}$$

$$(iv)\ y_\nu(\tilde{a}^*) = \frac{\frac{a_1'+a_2'-c_1'-c_2'}{4} + (\frac{a_1'+a_2'-c_1'-c_2'}{2})\ln\frac{1}{w}}{(a_1'+a_2'-c_1'-c_2')\ln\frac{1}{w}}. \tag{23}$$

Definition 11. The ranking function of the $T2IETFN$ $\tilde{a}^*$ which is the Euclidean distance is defined by

$$R(\tilde{a}^*) = \sqrt{\frac{1}{2}([x_\mu(\tilde{a}^*) - y_\mu(\tilde{a}^*)]^2 + [x_\nu(\tilde{a}^*) - y_\nu(\tilde{a}^*)]^2)}.$$

Therefore we can define ranking of Euclidean distance in $T2IETFN$.

Theorem 7. If $\tilde{a}^* = (a_1^*, b_1^*, c_1^*; w_1^*)(a_2^*, b_2^*, c_2^*; w_1^*)$ and $\tilde{z}^* = (a_3^*, b_3^*, c_3^*; w_2^*)(a_4^*, b_4^*, c_4^*; w_4^*)$ are two $T2IETFN$, so

$$i)\ If\ R(\tilde{a}^*) < R(\tilde{z}^*) \Longrightarrow \tilde{a}^* < \tilde{z}^*,$$

$$ii)\ If\ R(\tilde{a}^*) \approx R(\tilde{z}^*) \Longrightarrow \tilde{a}^* \approx \tilde{z}^*,$$

$$iii)\ If\ R(\tilde{a}^*) > R(\tilde{z}^*) \Longrightarrow \tilde{a}^* > \tilde{z}^*.$$

Example 5. If $\tilde{a}^* = [(2, 5, 8; 0.5)(1, 5, 9; 0.5), (0, 5, 10; 0.7)(-1, 5, 11; 0.7))]$ and $\tilde{z}^* = [(-1, 2, 5; 0.5)(-2, 2, 6; 0.5), (-3, 2, 7; 0.7)(-4, 2, 8; 0.7))]$ be two $T2IETFN$, then the using proposed method we get For $\tilde{a}^*$

$$x_\mu(\tilde{a}^*) = 0.08,\ y_\mu(\tilde{a}^*) = 0.86,$$

$$Then\ R(\tilde{a}^*) =\approx 8.88.$$

For $\tilde{z}^*$

$$x_\mu(\tilde{z}^*) = -0.03,\ y_\mu(\tilde{z}^*) = 0.86,$$

$$Then\ R(\tilde{z}^*) = \approx 6.77.$$

Using by Theorem 7, we have $R(\tilde{a}^*) > R(\tilde{z}^*) \Longrightarrow \tilde{a}^* > \tilde{z}^*$.

VII. Generalized type-2 intuitionistic exponential triangular fuzzy number transportation problem

Consider a type-2 intuitionistic exponential triangular fuzzy number with m sources and n destinations as
Minimize $\bigoplus_{i=1}^m \bigoplus_{j=1}^m c_{ij}x_{ij}$

$$subject\ to \begin{cases} \sum_{j=1}^n x_{ij} \approx a_i,\ for\ i=1,2,...,m \\ \sum_{i=1}^m x_{ij} \approx b_j,\ for\ j=1,2,...,n \\ x_{ij} \geq 0\ \forall\ i,j \end{cases} \tag{24}$$

where a_i is the approximate availability of the product at the ith source, b_i is the approximate demand of the product at the jth destination, c_{ij} is the approximate cost for transporting one unit of the product from the ith source to the jth destination and x_{ij} is the number of units of the product that should be transported from the ith source to jth destination taken as fuzzy decision variables.
If $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$ then the intuitionistic fuzzy transportation problem is said to be a balanced transportation problem, otherwise it is called an unbalanced.Let

$$c_{ij} = [(c_{ij1}, c_{ij2}, c_{ij3}; w)(c'_{ij1}, c_{ij2}, c'_{ij3}; w),$$

$$(c_{ij4}, c_{ij2}, c_{ij5}; w)(c'_{ij4}, c_{ij2}, c'_{ij5}; w)],$$

$$a_i = [(a_{i1}, a_{i2}, a_{i3}; w)(a'_{i1}, a_{i2}, a'_{i3}; w), (a_{i4}, a_{i2}, a_{i5}; w)(a'_{i4}, a_{i2}, a'_{i5}; w)],$$

and

$$b_j = [(b_{j1}, b_{j2}, b_{j3}; w)(b'_{j1}, b_{j2}, b'_{j3}; w), (b_{j4}, b_{j2}, b_{j5}; w)(b'_{j4}, b_{j2}, b'_{j5}; w)].$$

The steps to solve the above IFTP are as follows:
Step 1. Substituting the value of $c_{ij},\ a_i$ and b_j in (36), we get
Minimize
$\bigoplus_{i=1}^m \bigoplus_{j=1}^m [(c_{ij1}, c_{ij2}, c_{ij3}; w)(c'_{ij1}, c_{ij2}, c'_{ij3}; w), (c_{ij4}, c_{ij2}, c_{ij5}; w)(c'_{ij4}, c_{ij2}, c'_{ij5}; w)]x_{ij}$

$$subject\ to \begin{cases} \sum_{j=1}^n x_{ij} \approx [(a_{i1}, a_{i2}, a_{i3}; w)(a'_{i1}, a_{i2}, a'_{i3}; w), \\ (a_{i4}, a_{i2}, a_{i5}; w)(a'_{i4}, a_{i2}, a'_{i5}; w)], \\ \ for\ i=1,2,...,m \\ \sum_{i=1}^m x_{ij} \approx [(b_{j1}, b_{j2}, b_{j3}; w)(b'_{j1}, b_{j2}, b'_{j3}; w), \\ (b_{j4}, b_{j2}, b_{j5}; w)(b'_{j4}, b_{j2}, b'_{j5}; w)], \\ \ for\ j=1,2,...,n \\ x_{ij} \geq 0\ \forall\ i,j \end{cases} \tag{25}$$

Step 2. Now by the arithmetic operations and definitions presented in Section 4 and 6, (38) can be converted to crisp linear programming
Minimize

$H(\bigoplus_{i=1}^m \bigoplus_{j=1}^m [(x_{ij}c_{ij1}, x_{ij}c_{ij2}, x_{ij}c_{ij3}; w)(x_{ij}c'_{ij1}, x_{ij}c_{ij2}, x_{ij}c'_{ij3}; w), (x_{ij}c_{ij4}, x_{ij}c_{ij2}, x_{ij}c_{ij5}; w)(x_{ij}c'_{ij4}, x_{ij}c_{ij2}, x_{ij}c'_{ij5}; w)])$

$$subject\ to \begin{cases} H(\sum_{j=1}^n x_{ij}) = H([(a_{i1}, a_{i2}, a_{i3}; w)(a'_{i1}, a_{i2}, a'_{i3}; w), \\ (a_{i4}, a_{i2}, a_{i5}; w)(a'_{i4}, a_{i2}, a'_{i5}; w)]), \\ \ for\ i=1,2,...,m \\ H(\sum_{i=1}^m x_{ij}) = H([(b_{j1}, b_{j2}, b_{j3}; w)(b'_{j1}, b_{j2}, b'_{j3}; w), \\ (b_{j4}, b_{j2}, b_{j5}; w)(b'_{j4}, b_{j2}, b'_{j5}; w)]), \\ \ for\ j=1,2,...,n \\ x_{ij} \geq 0\ \forall\ i,j \end{cases} \tag{26}$$

Step 3. Find the optimal solution x_{ij} by solving the linear programming problem.
Step 4. Find the fuzzy optimal value by putting x_{ij} in $\bigoplus_{i=1}^m \bigoplus_{j=1}^m c_{ij}x_{ij}$.
Example 6. Consider a transportation problem with three origins and three destinations. The related costs are given in the following Table 1.
Minimize

$$[(3,4,5;0.5)(2,4,6;0.5),(1,4,7;0.5)(0,4,8;0.5)]x_{11}$$

$$\oplus[(3,4,7;0.2)(2,4,8;0.2),(1,4,9;0.2)(0,4,10;0.2)]x_{12}$$

$$\oplus[(4,5,6;0.3)(3,5,7;0.3),(2,5,8,0.3)(1,5,9;0.3)]x_{13}$$

$$\oplus[(3,4,6;0.6)(2,4,7;0.6),(1,4,8;0.6)(0,4,9;0.6)]x_{21}$$

$$\oplus[(5,6,7;0.6)(4,6,8;0.6),(3,6,9;0.6)(2,6,10;0.6)]x_{22}$$

$$\oplus[(5,6,8;0.4)(4,6,9;0.4),(3,6,10;0.4)(2,6,11;0.4)]x_{23}$$

$$\oplus[(3,5,8;0.7)(2,5,9;0.7),(1,5,10;0.7)(0,5,11;0.7)]x_{31}$$

$$\oplus[(4,5,8;0.8)(3,5,9;0.8),(2,5,10;0.8)(1,5,11;0.8)]x_{32}$$

$$\oplus[(4,6,7;0.2)(3,6,8;0.2),(2,6,9;0.2)(1,6,10;0.2)]x_{33}$$

$$subject\ to \begin{cases} x_{11}+x_{12}+x_{13} \approx [(4,5,7;0.6)(3,5,8;0.6), \\ (2,5,9;0.6)(1,5,10;0.6)] \\ x_{21}+x_{22}+x_{23} \approx [(4,5,8;0.5)(3,5,9;0.5), \\ (2,5,10;0.5)(1,5,11;0.5)] \\ x_{31}+x_{32}+x_{33} \approx [(6,7,8;0.8)(5,7,9;0.8), \\ (4,7,10;0.8)(3,7,11;0.8)] \\ x_{11}+x_{21}+x_{31} \approx [(5,7,9;0.9)(4,7,10;0.9), \\ (3,7,11;0.9)(2,7,12;0.9)] \\ x_{12}+x_{22}+x_{32} \approx [(3,4,8;0.8)(2,4,9;0.8), \\ (1,4,10;0.8)(0,4,11;0.8)] \\ x_{31}+x_{23}+x_{33} \approx [(4,5,8;0.8)(3,5,9;0.8), \\ (1,5,11;0.8)(0,5,12;0.8)] \\ x_{ij} \geq 0\ \forall\ i,j \end{cases}$$

With using step 2., transportation problem converted into crisp linear programming
Minimize

$$3.65x_{11} \oplus 5.09x_{12} \oplus 2.31x_{13} \oplus 3.14x_{21}$$

$\oplus 4.25x_{22} \oplus 6.39x_{23} \oplus 3.83x_{31} \oplus 4.11x_{32} \oplus 3.31x_{33}$

$$subject\ to \begin{cases} x_{11}+x_{12}+x_{13}=3.88 \\ x_{21}+x_{22}+x_{23}=4.37 \\ x_{31}+x_{32}+x_{33}=4.85 \\ x_{11}+x_{21}+x_{31}=5.05 \\ x_{12}+x_{22}+x_{32}=3.74 \\ x_{31}+x_{23}+x_{33}=2.07 \\ x_{ij}\geq 0\ \forall\ i,j \end{cases}$$

With using Lingo, we can solve the above crisp linear programming. We get $x_{11}=1.81$, $x_{12}=0$, $x_{13}=2.07$, $x_{21}=2.13$, $x_{22}=0$, $x_{23}=0$, $x_{31}=1.11$, $x_{32}=3.74$, $x_{33}=0$. So the minimum cost of transportation is 37.6991.

VIII. Application of System Failure using $T2IETFN$

Starting failure of an automobile depends on different facts. The facts are battery low charge, ignition failure and fuel supply failure. There are two sub-factors of each of the facts. The fault-tree of failure to start of the automobile is shown in the Fig.18. F_{fs}: represents the system failure to start of automobile,

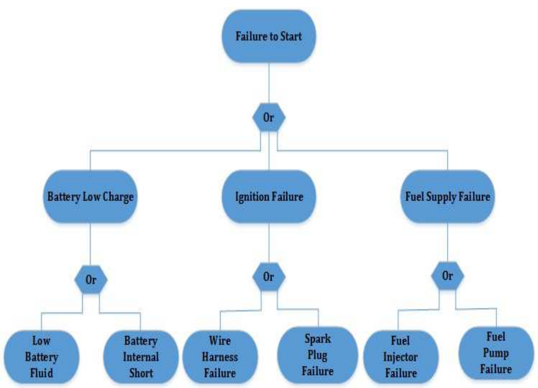

Fig. 9. Fault-tree of failure to start of an automobile

F_{blc}: represents the failure to start of automobile due to Battery Low Charge,
F_{if}: represents the failure to start of automobile due to Ignition Failure,
F_{fsf}: represents the failure to start of automobile due to Fuel Supply Failure,
F_{lbf}: represents the failure to start of automobile due to Low Battery Fluid,
F_{bis}: represents the failure to start of automobile due to Battery Internal Short,
F_{whf}: represents the failure to start of automobile due to Wire Harness Failure,
F_{spf}: represents the failure to start of automobile due to Spark Plug Failure,
F_{fif}: represents the failure to start of automobile due to Fuel Injector Failure,
F_{fpf}: represents the failure to start of automobile due to Fuel Pump Failure.

The intuitonistic fuzzy failure to start of an automobile can be calculated when the failures of the occurrence of basic fault events are known. Failure to start of an automobile can be evaluated by using the following steps:

$$\textbf{Step 1.} \begin{cases} F_{blc}=1\ominus(1\ominus F_{lbf})(1\ominus F_{bis}) \\ F_{if}=1\ominus(1\ominus F_{whf})(1\ominus F_{spf}) \\ F_{fsf}=1\ominus(1\ominus F_{fif})(1\ominus F_{fpf}) \end{cases} \tag{27}$$

$$\textbf{Step 2.}\ F_{fs}=1\ominus(1\ominus F_{blc})(1\ominus F_{if})(1\ominus F_{fsf}) \tag{28}$$

Example 7. Here we present numerical explanation of starting failure of the automobile using fault tree analysis with intuitionistic fuzzy failure rate. The components failure rates as $T2IETFN$ are given by

$$\begin{cases} F_{lbf}=[(0.03,0.04,0.06)(0.02,0.04,0.07), \\ (0.01,0.04,0.08)(0.00,0.04,0.09)], \\ F_{bis}=[(0.03,0.04,0.05)(0.02,0.04,0.06), \\ (0.01,0.04,0.07)(0.00,0.04,0.08)], \\ F_{whf}=[(0.03,0.05,0.07)(0.02,0.05,0.08), \\ (0.01,0.05,0.09)(0.00,0.05,0.1)], \\ F_{spf}=[(0.03,0.05,0.06)(0.02,0.05,0.07), \\ (0.01,0.05,0.08)(0.00,0.05,0.09)], \\ F_{fif}=[(0.04,0.05,0.06)(0.03,0.05,0.07), \\ (0.02,0.05,0.08)(0.01,0.05,0.09)], \\ F_{fpf}=[(0.04,0.05,0.07)(0.03,0.05,0.08), \\ (0.02,0.05,0.09)(0.01,0.05,0.1)]. \end{cases} \tag{29}$$

In the step 2 (using Eq.37), we obtain the failure to start of the automobile. The fuzzy failure to start of an automobile (Fig.18) is represented by the following $T2IETFN$

$$F_{fs}=[(0.18412,0.24935,0.31755)(0.13214,0.24935,0.36004),$$

$$(0.07744,0.24935,0.40031)(0.0199,0.24935,0.43835)]$$

So the failure to start of the automobile is about 0.24935, with tolerance level of acceptance $[0.18412,0.40031]$ and tolerance level of rejection $[0.13214,0.4383]$.

Example 8. system failure data used in this example are adopted from Shaw and Roy[22]

$$\begin{cases} F_{lbf}=[(0.03,0.04,0.05)(0.02,0.04,0.06), \\ (0.01,0.04,0.07)(0.00,0.04,0.08], \\ F_{bis}=[(0.03,0.05,0.06)(0.02,0.05,0.07), \\ (0.01,0.05,0.08)(0.00,0.05,0.09)], \\ F_{whf}=[(0.02,0.03,0.04)(0.01,0.03,0.05), \\ (0.00,0.03,0.06)(0.001,0.03,0.07)], \\ F_{spf}=[(0.04,0.06,0.07)(0.02,0.06,0.09), \\ (0.01,0.06,0.1)(0.00,0.06,0.11)], \\ F_{fif}=[(0.06,0.07,0.09)(0.04,0.07,0.1), \\ (0.03,0.07,0.11)(0.02,0.07,0.12)], \\ F_{fpf}=[(0.05,0.07,0.08)(0.03,0.07,0.09), \\ (0.02,0.07,0.1)(0.01,0.07,0.11)]. \end{cases}$$

In the step 2 (using Eq.37), we obtain the failure to start of the automobile is represented by the following $T2IETFN$

$$F_{fs}=[(0.20952,0.28078,0.33253)(0.13233,0.28078,0.38105),$$

$(0.07763, 0.28078, 0.42021)(0.03077, 0.28078, 0.45728)]$

With using median between $[\mu_{\tilde{F_{fs}}^{L}}(x), \mu_{\tilde{F_{fs}}^{U}}(x), \nu_{\tilde{F_{fs}}^{L}}(x), \nu_{\tilde{F_{fs}}^{U}}(x)]$, we have:
The failure to start of the automobile is about 0.28078, with tolerance level of acceptance $[0.143575, 0.37637]$ and tolerance level of rejection $[0.08155, 0.419165]$.

IX. CONCLUSION

$T2IETFN$ has many advantages over type-1 fuzzy sets because their membership functions are themselves fuzzy, making it possible to model and minimize the effect of uncertainty in type-1 intuitionistic fuzzy systems and we can easily find the membership, non-membership and hesitation degree. Basic generalized exponential triangular intuitionistic fuzzy numbers formulated of (α, β)-cut methods and ranking of $T2IETFN$ play an important role in intuitionistic fuzzy decision making problem. (α, β)-cut of fuzzy number is very important in defining total ordering on the class of intuitionistic fuzzy numbers. It means, we can compare any two intuitionistic fuzzy numbers using their alpha-beta cuts. This proposed ranking method can apply for both kind of fuzzy number and flexible to the researchers in the ranking index of their attitudinal analysis.
We have taken the intuitionistic fuzzy failure to start of an automobile as known basic fault events using Type-2 intuitionistic exponential triangular fuzzy number. The grade of a membership function indicates a subjective degree of preference of a decision maker within a given tolerance and grade of a non-membership function indicates a subjective degree of negative response of a decision maker within a given tolerance. The proposed technique can be suitably utilized to solve the start of an automobile problem.

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Table (1): Input data for IFTP.

	D_1	D_2	D_3	Availability (a_i)	$Demand(b_i)$
S_1	[(3,4,5;0.5)(2,4,6;0.5),(1,4,7;0.5)(0,4,8;0.5)],	[(3,4,7;0.2)(2,4,8;0.2),(1,4,9;0.2)(0,4,10;0.2)],	[(4,5,6;0.3)(3,5,7;0.3),(2,5,8,0.3)(1,5,9;0.3)],	[(4,5,7;0.6)(3,5,8;0.6),(2,5,9;0.6)(1,5,10;0.6)],	[(5,7,9;0.9)(4,7,10;0.9),(3,7,11;0.9)(2,7,12;0.9)],
S_2	[(3,4,6;0.6)(2,4,7;0.6),(1,4,8;0.6)(0,4,9;0.6)],	[(5,6,7;0.6)(4,6,8;0.6),(3,6,9;0.6)(2,6,10;0.6)],	[(5,6,8;0.4)(4,6,9;0.4),(3,6,10;0.4)(2,6,11;0.4)],	[(4,5,8;0.5)(3,5,9;0.5),(2,5,10;0.5)(1,5,11;0.5)],	[(3,4,8;0.8)(2,4,9;0.8),(1,4,10;0.8)(0,4,11;0.8)],
S_3	[(3,5,8;0.7)(2,5,9;0.7),(1,5,10;0.7)(0,5,11;0.7)]	[(4,5,8;0.8)(3,5,9;0.8),(2,5,10;0.8)(1,5,11;0.8)]	[(4,6,7;0.2)(3,6,8;0.2),(2,6,9;0.2)(1,6,10;0.2)]	[(6,7,8;0.8)(5,7,9;0.8),(4,7,10;0.8)(3,7,11;0.8)]	[(4,5,8;0.8)(3,5,9;0.8),(1,5,11;0.8)(0,5,12;0.8)]

Table (2): A comparison of the failure to start of the automobile

Approaches	failure to start	tolerance acceptance	tolerance rejection
Shaw and Roy[29]	0.2807824	[0.2095175, 0.3325252]	[0.1323264,0.3008815]
Proposed approach	0.2807824	[0.143575,0.37637]	[0.08155,0.419165]
Compare	0	$+89.25$	-35.74