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TOPSIS-WAA method based on a covering-based fuzzy rough set: An application to rating problem

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ABSTRACT

In this paper, we firstly study two pairs of covering-based fuzzy rough set models and propose the TOPSIS-WAA method based on a covering-based fuzzy rough set. Subsequently, we design a rating scheme based on a multi-criteria decision-making method in a finite fuzzy covering approximation space. The rating scheme is implemented based on the preset subjective ratio and the calculated objective ranking of all alternatives. Furthermore, we use the relevant data of some customers of Industrial and Commercial Bank of China (ICBC) to illustrate the feasibility of our method. At the same time, two test rules are used to verify the validity of the proposed method. Moreover, we compare five different decision-making methods with our method to demonstrate the superiority of our method. Finally, the performance of our method is verified from the perspectives of the best alternative and the optimal ranking.

1. Introduction

The data in the information era presents large and complex features. The complexity of these data has caused a lot of problems for researchers. In order to deal with these problems, some scholars have proposed various processing methods [2,7,26]. In this paper, we focus on how to study new multi-criteria decision-making (MCDM) methods to cope with increasingly complex practical problems. In general, MCDM information mainly involves three aspects, namely, objects, criteria and evaluation values. The quantity information of objects and criteria determines the volume of data. This means that as the number of objects and criteria increases, the capacity of data that can be collected also increases. The diversities of evaluation value information determine the diversities of processable and analyzable data. This means we need to use different decision-making methods to solve problems in different environments. This paper considers the rating problem in a finite fuzzy covering approximation space (FCAS).

In real life, some businesses or companies often provide some special services to their new and regular customers. For example, banks provide some customers with preferential services. However, due to the limited resources, the number of customer groups enjoying these preferential services will also be limited. Moreover, the level of preferential services enjoyed by these customers will also be different. In other words, these customers can get the preferential services of businesses or companies, but the level of available service is different, that is, the differential treatment of the services. For example, suppose a bank has 1,000 preferential services that will be provided to customers who register within a certain period of time.

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But the level of these services can be divided into several levels and there are certain differences in quantity. In short, these 1,000 customers will enjoy different levels of preferential services, that is, these customers will enjoy different treatment. In fact, we can also regard this differential treatment as a priority issue. That is to say, the higher the customer's ranking or priority, the higher the rating, and the better the service. Therefore, in this paper, we design a rating scheme based on an MCDM method in a finite FCAS to deal with this type of problem. At the same time, we propose a TOPSIS-WAA method based on a covering-based fuzzy rough set (CFRS). This ranking decision-making method is the core of this scheme and can provide an objective sorting information for the rating work. Before describing this scheme, we firstly review the relevant research of TOPSIS methods, the weighted arithmetic average (WAA) operator method and the CFRS theory.

1.1. The TOPSIS method, the WAA operator method and the covering-based fuzzy rough sets

On the one hand, we introduce the methodological ideological basis of the TOPSIS-WAA method in a finite FCAS.

- The core idea of TOPSIS methods [12] is to find the optimal solution that is close to the positive ideal solution but far from the negative ideal solution. In recent years, some scholars have developed a large number of novel TOPSIS methods [3,15] in various environments and applied TOPSIS methods [34,39] to realistic problems.
- In addition to the TOPSIS method, the WAA operator method plays an important role in traditional decision theory. The WAA operator method is a type of aggregation operator method [28]. The core of this method is a weighted arithmetic average operator. Based on the basic idea of WAA operator methods, the final value of each alternative can be obtained by using the weighted data. The WAA operator method is not only easy to understand in theory, but also has extensive application value [8,16].

On the other hand, we introduce the theoretical development of the CFRS model in a finite FCAS. Rough set theory [19] has been widely promoted and utilized [10,27] since its appearance. CFRS theory is an important branch of rough set theory, which is mainly produced by the combination of the covering-based rough set (CRS) theory [31,32,35] and the fuzzy rough set (FRS) theory [1,9,24]. Recently, some scholars have made important contributions to CFRS theory. Based on different fuzzy coverings, D'eer et al. [6] constructed different neighborhood operators and studied their relationships. Subsequently, D'eer and Cornelis [5] conducted a comprehensive study on fuzzy covering-based rough set models. Besides, Ma [17] proposed the concept of fuzzy β-coverings and explored two FRS models. Based on fuzzy β-coverings, Yang and Hu [29] also developed several new models.

Based on the above three theories, we develop an MCDM method in a finite FCAS to solve the rating problem. Afterwards, the following motivations of this paper will be described in detail.

1.2. The motivation of this paper

There are some related studies on MCDM methods in a finite FCAS. For example, Jiang et al. [13] constructed four types of new variable-precision fuzzy rough set models and explored the application of multi-attribute decision-making method in the selection of financial products. Zhan et al. [36] proposed a multi-criteria group decision making method by using a new multi-granularity fuzzy rough set model. In addition, based on the rough set theory and the MCDM theory, some scholars have also studied plenty of new decision-making methods [21,22,40]. However, the above-mentioned several decision-making methods are not suitable for our proposed rating scheme. The reason is that the core of the rating scheme designed in this paper is the MCDM method in a finite FCAS. In order to better implement this scheme, we develop a new MCDM method. Moreover, we explain the specific research motivations of this paper as follows:

- In real life, the limited resources of commercial services result in a limited number of customers that can enjoy services. At the same time, the different levels of services require merchants to classify customers accordingly. Hence, how to grade a certain number of customers act as a valuable topic. In view of this, this paper uses a pre-set subjective ratio (that is, a quantitative ratio of all levels) and an objective ranking of all customers to form an effective rating scheme. Correspondingly, the subjective ratio can be determined by the merchant itself (or the decision-maker), and the objective ranking of all customers is determined by the new decision method proposed in this paper.
- The core of our proposed rating scheme is an MCDM method in a finite FCAS. However, some common MCDM methods may fail in a finite FCAS (see Example 3.1 in this paper). In order to better implement our rating scheme, we develop a new MCDM method by combining the TOPSIS method, the WAA operator method with CFRS theory.

1.3. The organization of this paper

The organization of the paper is listed below. In Section 2, some basic knowledge related to fuzzy logical operators and fuzzy neighborhood operators will be described. Two pairs of CFRS models are proposed in Section 3. In Section 4, we give an MCDM method in a finite FCAS. In Section 5, a numerical example to explain the effectiveness of our scheme and two test rules are used to verify the validity of the proposed method. Further, we compare five different decision-making methods with our method in Section 6. Finally, we verify the performance of our method from the perspectives of the optimal alternative and the optimal sorting scheme in Section 7.

2. Preliminary

In this section, some basic knowledge related to fuzzy logical operators and fuzzy neighborhood operators will be described.

2.1. Fuzzy logical operators

As we all know, a t-norm $\mathscr{T}$ is a generalization of Zadeh operators. Essentially, a t-norm operator is a mapping on [0,1] that satisfies the commutative law, conjunction law, monotonicity and boundary condition. Moreover, based on a continuous t-norm $\mathscr{T}$, we can obtain an R-implicator $\mathscr{I}$. The R-implicator is defined as $\mathscr{I}(m,n) = \bigvee\{\delta \in [0,1] : \mathscr{T}(m,\delta) \leqslant n\}, m,n \in [0,1]$. These two operators will be used in the research of this paper. Readers can learn more about these two operators from [18,20].

2.2. Covering-based fuzzy neighborhood operators

From [6], we know that the relevant information of fuzzy coverings and covering-based fuzzy neighborhood operators.

Definition 2.1. Assume that $\mathscr{F}(W)$ is a family of fuzzy subsets on W. For every $e \in W$, if there is a $H \in \mathbf{H}$ such that $H(e) = 1$, then $\mathbf{H} = \{H_\lambda \in \mathscr{F}(W) : H_\lambda \neq \emptyset, \lambda \in \nabla\}$ is called a fuzzy covering (FC), here ∇ is an index set. $(W, \mathbf{H})$ is called a finite FCAS.

Note that the fuzzy covering is finite in this paper. According to a finite FC $\mathbf{H}$, we have the following definition:

Definition 2.2. Let $\mathbf{H}$ be a finite FC, for every $e,f \in W$, four fuzzy covering-based fuzzy neighborhood operators are expressed as follows:

$$\begin{aligned} N_1^{\mathbf{H}}(e)(f) &= \inf_{H\in\mathbf{H}} \mathscr{I}(H(e),H(f)), \\ N_2^{\mathbf{H}}(e)(f) &= \sup_{H\in md(\mathbf{H},e)} \mathscr{T}(H(e),H(f)), \\ N_3^{\mathbf{H}}(e)(f) &= \inf_{H\in MD(\mathbf{H},e)} \mathscr{I}(H(e),H(f)), \\ N_4^{\mathbf{H}}(e)(f) &= \sup_{H\in\mathbf{H}} \mathscr{T}(H(e),H(f)), \end{aligned} \tag{2-1}$$

where $md(\mathbf{H},e)$ and $MD(\mathbf{H},e)$ are called the fuzzy minimum description and fuzzy maximum description of e, respectively

In [6], D'eer et al. explored the properties and relationships of these four fuzzy neighborhood operators. The operators $N_1^{\mathbf{H}}$ and $N_3^{\mathbf{H}}$ are reflexive and $\mathscr{T}$-transitive. The operator $N_2^{\mathbf{H}}$ is reflexive. The operator $N_4^{\mathbf{H}}$ is reflexive and symmetric. Moreover, from [6], we have $N_1^{\mathbf{H}} \leqslant N_2^{\mathbf{H}} \leqslant N_4^{\mathbf{H}}$[1] and $N_1^{\mathbf{H}} \leqslant N_3^{\mathbf{H}} \leqslant N_4^{\mathbf{H}}$[2].

3. Two pairs of covering-based fuzzy rough set models

From the study of a large number of MCDM research literatures, we find that MCDM methods can be applied to a variety of complex environments. A finite FCAS is a complex and real environment. However, there are few applications of MCDM methods in a finite FCAS. Moreover, there are some decision-making problems in a finite FCAS. For example, there exists a failure situation for the TOPSIS method and the WAA operator method in a finite FCAS. The following example will show this problem.

Example 3.1. Let $W = \{k_i | i = 1,2,\cdots,10\}$ be 10 alternatives and $\mathbf{H} = \{H_j | j = 1,2,\cdots,6\}$ be 6 criteria. In addition, $H_j(k_i)$ indicates a fuzzy score of the alternative k_i with respect to the criterion H_j, where $H_j(k_i) \in [0,1]$. The MCDM matrix with fuzzy information is shown as Table 1.

Note : In Table 1, based on the TOPSIS method [12] and the WAA operator method [28], the ranking results of ten clients are identical, that is, $k_1 \approx k_2 \approx k_3 \approx k_4 \approx k_5 \approx k_6 \approx k_7 \approx k_8 \approx k_9 \approx k_{10}$. This phenomenon illustrates the TOPSIS method and the WAA operator method may fail in a finite FCAS.

In light of Example 3.1, we have the following question:

Question : In a finite FCAS, how to combine the advantages of the TOPSIS method with the WAA operator method to provide a decision-maker with the optimal ranking scheme and the best alternative?.

In light of the above problems, we develop a new TOPSIS-WAA method in a finite FCAS. Before that, we will introduce two pairs of CFRS models.

[1] $N_1^{\mathbf{H}} \leqslant N_2^{\mathbf{H}} \leqslant N_4^{\mathbf{H}}$ is defined as $N_1^{\mathbf{H}}(e)(f) \leqslant N_2^{\mathbf{H}}(e)(f) \leqslant N_4^{\mathbf{H}}(e)(f)$ for every $e,f \in W$.

[2] $N_1^{\mathbf{H}} \leqslant N_3^{\mathbf{H}} \leqslant N_4^{\mathbf{H}}$ is defined as $N_1^{\mathbf{H}}(e)(f) \leqslant N_3^{\mathbf{H}}(e)(f) \leqslant N_4^{\mathbf{H}}(e)(f)$ for every $e,f \in W$.

Table 1
The MCDM matrix with fuzzy information.

$W/\mathbf{H}$	H_1	H_2	H_3	H_4	H_5	H_6
k_1	1	0.2	0.5	0.9	0.5	0.8
k_2	0.5	0.7	1	0.4	0.6	0.7
k_3	0.4	0.8	0.6	0.8	1	0.3
k_4	0.2	1	0.7	0.7	0.8	0.5
k_5	0.8	0.4	0.8	0.6	0.3	1
k_6	0.9	0.3	1	0.4	0.4	0.9
k_7	0.3	0.9	0.5	0.9	1	0.3
k_8	0.4	0.8	0.6	0.8	0.3	1
k_9	0.2	1	0.5	0.9	0.6	0.7
k_{10}	0.6	0.6	0.4	1	0.3	1
Weight	0.25	0.25	0.15	0.15	0.1	0.1

Definition 3.2. Let $(W,\mathbf{H})$ be a finite FCAS and $\mathscr{I}$ and $\mathscr{T}$ be an R-implicator and a continuous t-norm, respectively. For any $H\in\mathscr{F}(W), e\in W$, the pairs $\left(\underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{I}}(H),\overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{T}}(H)\right)$ and $\left(\underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{I}}(H),\overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{T}}(H)\right)$ are defined as follows:

$$\underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{I}}(H)(e)=\bigvee_{f\in W}\mathscr{T}\left(N_\lambda^{\mathbf{H}}(f)(e),\bigwedge_{g\in W}\mathscr{I}\left(N_\lambda^{\mathbf{H}}f\right)(g),H(g)\right)), \tag{3-1}$$

$$\overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{T}}(H)(e)=\bigwedge_{f\in W}\mathscr{I}\left(N_\lambda^{\mathbf{H}}(f)(e),\bigvee_{g\in W}\mathscr{T}\left(N_\lambda^{\mathbf{H}}(f)(g),H(g)\right)\right), \tag{3-2}$$

$$\underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{I}}(H)(e)=\bigwedge_{f\in W}\mathscr{I}\left(N_\lambda^{\mathbf{H}}(f)(e),\bigwedge_{g\in W}\mathscr{I}\left(N_\lambda^{\mathbf{H}}(f)(g),H(g)\right)\right), \tag{3-3}$$

$$\overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{T}}(H)(e)=\bigvee_{f\in W}\mathscr{T}\left(N_\lambda^{\mathbf{H}}(f)(e),\bigvee_{g\in W}\mathscr{T}\left(N_\lambda^{\mathbf{H}}(f)(g),H(g)\right)\right). \tag{3-4}$$

The operators $\underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{I}}$, $\overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{T}}$, $\underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{I}}$ and $\overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{T}}$ are called $N_\lambda\mathscr{T}\mathscr{I}$-lower, $N_\lambda\mathscr{I}\mathscr{T}$-upper, $N_\lambda\mathscr{I}\mathscr{I}$-lower and $N_\lambda\mathscr{T}\mathscr{T}$-upper fuzzy rough approximation operators, respectively. The pairs $\left(\underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{I}},\overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{T}}\right)$ and $\left(\underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{I}},\overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{T}}\right)$ are called $(N_\lambda\mathscr{T}\mathscr{I},N_\lambda\mathscr{I}\mathscr{T})$-CFRS model (the first pair) and $(N_\lambda\mathscr{I}\mathscr{I},N_\lambda\mathscr{T}\mathscr{T})$-CFRS model (the second pair) on $(W,\mathbf{H})$, respectively. Moreover, for every $H\in\mathscr{F}(W)$, $\left(\underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{I}}(H),\overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{T}}(H)\right)$ and $\left(\underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{I}}(H),\overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{T}}(H)\right)$ are denoted $(N_\lambda\mathscr{T}\mathscr{I},N_\lambda\mathscr{I}\mathscr{T})$-CFRS model and $(N_\lambda\mathscr{I}\mathscr{I},N_\lambda\mathscr{T}\mathscr{T})$-CFRS model of H on $(W,\mathbf{H})$, respectively. Note that the parameter λ can take values 1, 2, 3 and 4, respectively

Next we continue to study the degradations of the above two models.

Remark 3.3. In [6], D'eer et al. showed that a fuzzy neighborhood operator and a fuzzy binary relationship are equivalent in some conditions. Therefore, the above two pairs of models have the following degenerated forms.

(i) If a fuzzy neighborhood operator $N_\lambda^{\mathbf{H}}$ is replaced by a fuzzy binary relation R, for every $H\in\mathscr{F}(W)$ and $e\in W$, $\left(\underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{I}}(H),\overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{T}}(H)\right)$ and $\left(\underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{I}}(H),\overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{T}}(H)\right)$ can be degenerate into the following forms:

$$\underline{F}_{R,\mathscr{T},\mathscr{I}}(H)(e)=\bigvee_{f\in W}\mathscr{T}\left(R(f,e),\bigwedge_{g\in W}\mathscr{I}(R(f,g),H(g))\right), \tag{3-5}$$

$$\overline{F}_{R,\mathscr{I},\mathscr{T}}(H)(e)=\bigwedge_{f\in W}\mathscr{I}\left(R(f,e),\bigvee_{g\in W}\mathscr{T}(R(f,g),H(g))\right), \tag{3-6}$$

$$\underline{F}_{R,\mathscr{I},\mathscr{I}}(H)(e)=\bigwedge_{f\in W}\mathscr{I}\left(R(f,e),\bigwedge_{g\in W}\mathscr{I}(R(f,g),H(g))\right), \tag{3-7}$$

$$\overline{F}_{R,\mathscr{T},\mathscr{T}}(H)(e)=\bigvee_{f\in W}\mathscr{T}\left(R(f,e),\bigvee_{g\in W}\mathscr{T}(R(f,g),H(g))\right). \tag{3-8}$$

(ii) If R is symmetric and $\mathscr{T}$-transitive, then both pairs can degenerate into the following form:

$$\underline{R}_{\mathscr{I}}(H)(e) = \bigwedge_{f \in W} \mathscr{I}(R(e,f), H(f)), \tag{3-9}$$

$$\overline{R}_{\mathscr{T}}(H)(e) = \bigvee_{f \in W} \mathscr{T}(R(e,f), H(f)). \tag{3-10}$$

We obtain more information about the model $(\underline{R}_{\mathscr{I}}, \overline{R}_{\mathscr{T}})$ from [20].

We also explore the relationship between models $\left(\underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{I}}, \overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{T}}\right)$ and $\left(\underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{I}}, \overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{T}}\right)$ as follows.

Remark 3.4. In Remark 3.3, we find that the covering-based fuzzy rough set model can be degenerated into a fuzzy rough set model based on a fuzzy binary relation. Moreover, the semantics of fuzzy rough set models based on a fuzzy binary relation can be explained from the relationship between an element and a set. In other words, the fuzzy rough upper approximation and fuzzy rough lower approximation of a fuzzy set can represent the maximum degree and minimum degree to which an element belongs to this fuzzy set.

In fact, the difference between the two fuzzy rough set models is that they have different fuzzy binary relations. The fuzzy rough set model based on a fuzzy binary relation is established based on the binary relationship between objects. Thus, in practice, researchers need to obtain the binary relationship between objects. However, in real life, we often obtain a binary relationship between object and criterion rather than a binary relationship between objects. Correspondingly, the covering-based fuzzy rough set model is based on the fuzzy binary relationship between object and criterion. Furthermore, the binary relationship between object and criterion is represented by an object-criteria information table, also called a multi-criteria information table. Moreover, in the multi-criteria information table, for any object, if the conditions in Definition 2.1 are met, then the multi-criteria information table is a finite FCAS. In light of this, based on the multi-criteria information table, we can use the fuzzy neighborhood operator to obtain the binary relationship between objects. Therefore, the covering-based fuzzy rough set model is a general fuzzy rough set model in a finite FCAS (or multi-criteria information table). It can also be said that the covering-based fuzzy rough set model is a special fuzzy rough set model based on a fuzzy binary relation.

Therefore, for the covering-based fuzzy rough set model, the fuzzy rough upper approximation and fuzzy rough lower approximation of a fuzzy set can represent the maximum and minimum degrees to which an element belongs to this fuzzy set.

Theorem 3.5. *Let* $(W, \mathbf{H})$ *be a finite FCAS. For every* $H \in \mathscr{F}(W)$, *we have* $\underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{I}}(H) \subseteq \underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{I}}(H) \subseteq H \subseteq \overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{T}}(H) \subseteq \overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{T}}(H)$.

Proof. For every $H \in \mathscr{F}(W)$ and $e \in W$,

$$\begin{aligned}
\underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{I}}(H)(e) &= \bigwedge_{f \in W} \mathscr{I}\left(N_\lambda^{\mathbf{H}}(f)(e), \bigwedge_{g \in W} \mathscr{I}\left(N_\lambda^{\mathbf{H}}(f)(g), H(g)\right)\right) \\
&\leqslant \mathscr{I}\left(N_\lambda^{\mathbf{H}}(e)(e), \bigwedge_{g \in W} \mathscr{I}\left(N_\lambda^{\mathbf{H}}(e)(g), H(g)\right)\right) \\
&= \mathscr{I}\left(1, \bigwedge_{g \in W} \mathscr{I}\left(N_\lambda^{\mathbf{H}}(e)(g), H(g)\right)\right) \\
&= \bigwedge_{g \in W} \mathscr{I}\left(N_\lambda^{\mathbf{H}}(e)(g), H(g)\right) \\
&= \mathscr{T}\left(1, \bigwedge_{g \in W} \mathscr{I}\left(N_\lambda^{\mathbf{H}}(e)(g), H(g)\right)\right) \\
&= \mathscr{T}\left(N_\lambda^{\mathbf{H}}(e)(e), \bigwedge_{g \in W} \mathscr{I}\left(N_\lambda^{\mathbf{H}}(e)(g), H(g)\right)\right) \\
&\leqslant \bigvee_{f \in W} \mathscr{T}\left(N_\lambda^{\mathbf{H}}(f)(e), \bigwedge_{g \in W} \mathscr{I}\left(N_\lambda^{\mathbf{H}}(f)(g), H(g)\right)\right) \\
&= \underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{I}}(H)(e) \\
&\leqslant H(e).
\end{aligned}$$

Hence, $\underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{I}}(H) \subseteq \underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{I}}(H) \subseteq H$. Likewise, $H \subseteq \overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{T}}(H) \subseteq \overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{T}}(H)$. This completes the proof.

Remark 3.6. [5] Let A and B be two fuzzy neighborhood operators on W. For any $H \in \mathscr{F}(W)$, if $A \leqslant B$, then $\underline{F}_{B,\mathscr{I},\mathscr{I}}(H) \subseteq \underline{F}_{A,\mathscr{I},\mathscr{I}}(H) \subseteq \overline{F}_{A,\mathscr{T},\mathscr{T}}(H) \subseteq \overline{F}_{B,\mathscr{T},\mathscr{T}}(H)$.

Theorem 3.5 demonstrates the relationship between the model $\left(\underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{I}}, \overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{T}}\right)$ and the model $\left(\underline{F}_{N_\lambda^{\mathbf{H}},\mathscr{I},\mathscr{I}}, \overline{F}_{N_\lambda^{\mathbf{H}},\mathscr{T},\mathscr{T}}\right)$. In fact, by taking different fuzzy neighborhood operators (i.e., $N_\lambda^{\mathbf{H}}, \lambda = 1,2,3,4$), these two pairs of models will also derive different model forms. Besides, according to Theorem 3.5 and Remark 3.6, for every $H \in \mathscr{F}(W)$, the relationships among these derived models are shown as the following Fig. 1:

In Fig. 1, $\rightarrow$ represents $\subseteq$. Moreover, we can see that the model $\left(\underline{F}_{N_1^{\mathbf{H}},\mathscr{T},\mathscr{I}}, \overline{F}_{N_1^{\mathbf{H}},\mathscr{I},\mathscr{T}}\right)$ is a relatively compact model. Next we apply this model to MCDM.

4. A method to MCDM based on $(N_1\mathcal{TI}, N_1\mathcal{IT})$-CFRS models

In this section, we propose an MCDM method in a finite FCAS. This method is a combination of the TOPSIS method, the WAA operator method and CFRS theory, which can effectively solve the problem of the TOPSIS method and the WAA operator method in a finite FCAS.

4.1. Background description

How to develop a new MCDM method in a finite FCAS is the main research interest of this paper. Generally speaking, an MCDM problem can be represented by a multi-criteria information table. At the same time, in the multi-criteria information table, for any object, if the conditions in Definition 2.1 are met, then the multi-criteria information table can be called a finite FCAS. Therefore, how to extend the MCDM methods in a finite FCAS will be a good research direction. In light of this, we develop a new MCDM method in a finite FCAS in the following section. Besides, in order to improve readability, we extract some MCDM information (Table 2) as follows:

In the following section, we will describe the steps of our MCDM method in detail.

4.2. Method description

Firstly, a multi-criteria information table (Table 3) that can form a finite FCAS is as follows:

From Table 3, the score of the alternative k_i on the criterion H_j is h_{ij} $(H_j(k_i))$, where $h_{ij} \in [0,1]$. Furthermore, For every $k \in W$, there is a $H \in \mathbf{H}$ such that $H(k) = 1$. $\mathbf{H}$ forms a finite FC and $(W, \mathbf{H})$ forms a finite FCAS.

Note : our method is a combination of the TOPSIS method, the WAA operator method and CFRS models. Therefore, we need a suitable CFRS model. This paper applies the model $\left(\underline{F}_{N_1^{\mathbf{H}},\mathscr{T},\mathscr{I}}, \overline{F}_{N_1^{\mathbf{H}},\mathscr{I},\mathscr{T}}\right)$ to MCDM. The reason is that this model is relatively compact (Fig. 1). Of course, other models that appear in this paper can also be applied to MCDM. We will implement our method on this information matrix.

In Table 3, each criterion can form a fuzzy set as follows:

$$H_j = \sum_{i=1}^{n} \frac{h_{ij}}{k_i}, j \in \{1,2,\cdots,m\}. \tag{4-1}$$

We can use the model $\left(\underline{F}_{N_1^{\mathbf{H}},\mathscr{T},\mathscr{I}}, \overline{F}_{N_1^{\mathbf{H}},\mathscr{I},\mathscr{T}}\right)$ to calculate the lower and upper approximation sets for each criterion H_j as follows:.

$$\underline{F}_{N_1^{\mathbf{H}},\mathscr{T},\mathscr{I}}(H_j) = \sum_{i=1}^{n} \frac{u_{ij}}{k_i}, j \in \{1,2,\cdots,m\}, \tag{4-2}$$

$$\overline{F}_{N_1^{\mathbf{H}},\mathscr{I},\mathscr{T}}(H_j) = \sum_{i=1}^{n} \frac{v_{ij}}{k_i}, j \in \{1,2,\cdots,m\}. \tag{4-3}$$

By the formulas (4–2) and (4–3), we can obtain the lower and upper approximation information matrices with fuzzy information, respectively.

We will use the fuzzy information matrix in Table 4 and Table 5, respectively, for the following descriptions of methods.

Based on Table 4 (Table 5), the positive ideal point $u_{j,+}$ $(v_{j,+})$ and the negative ideal point $u_{j,-}$ $(v_{j,-})$ of each criterion H_j are expressed as the following formulas:

$$u_{j,+}(v_{j,+}) = \begin{cases} \max\limits_{1 \leqslant i \leqslant n} u_{ij}\left(\max\limits_{1 \leqslant i \leqslant n} v_{ij}\right), & H_j \in C, j \in \{1,2,\cdots,m\}, \\ \min\limits_{1 \leqslant i \leqslant n} u_{ij}\left(\min\limits_{1 \leqslant i \leqslant n} v_{ij}\right), & H_j \in D, j \in \{1,2,\cdots,m\}, \end{cases} \tag{4-4}$$

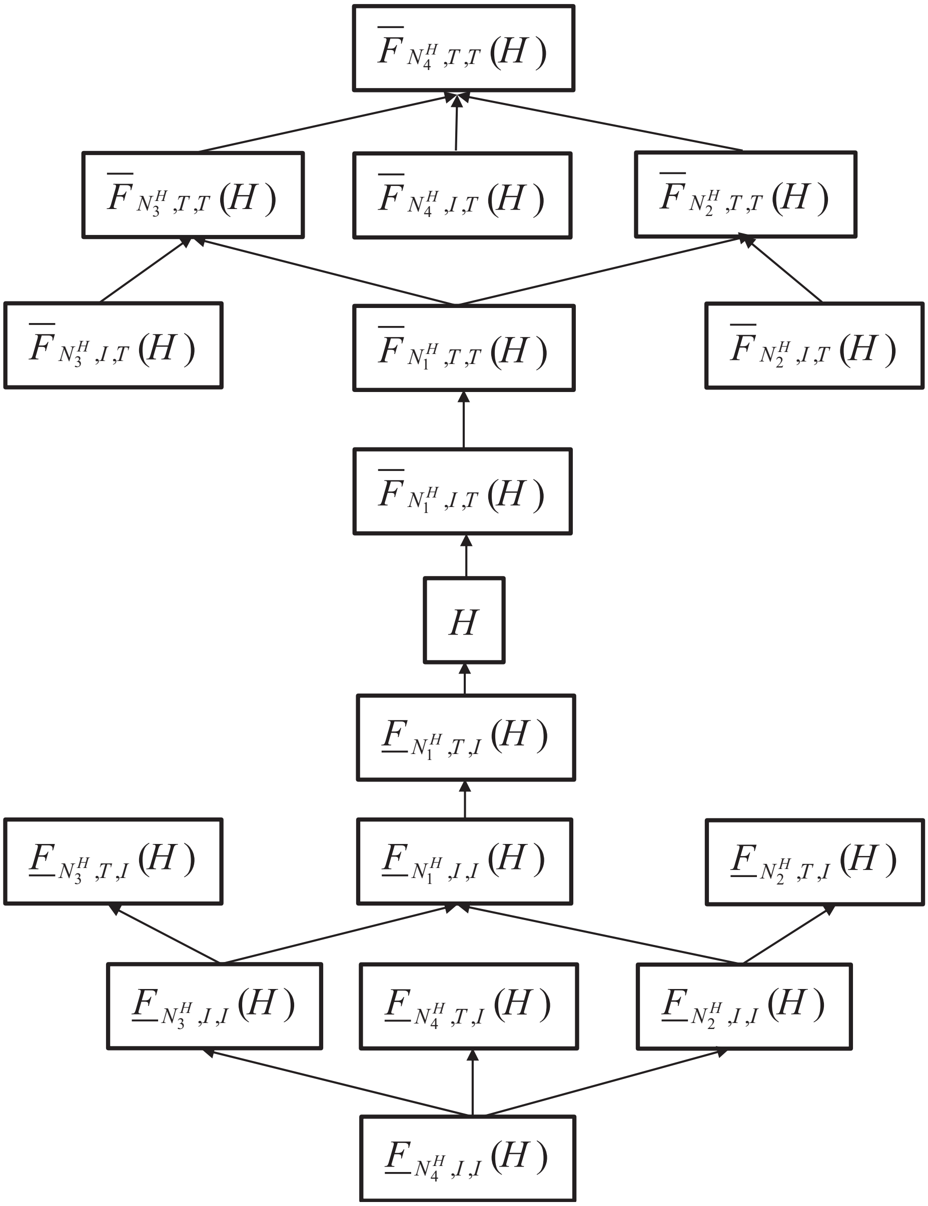

Fig. 1. The relationships among different models.

Table 2
The MCDM information table.

Designation	Information
Number of criteria	m
Number of alternatives	n
Set of alternatives	$W = \{k_i \mid i = 1, 2, \cdots, n\}$
Set of criteria	$\mathbf{H} = \{H_j \mid j = 1, 2, \cdots, m\}$
Evaluation value (score)	$h_{ij}\ (H_j(k_i))$
Weight vector	$T = \{t_j \mid j = 1, 2, \cdots, m\}$, $0 \leqslant t_j \leqslant 1$, $\sum_{j=1}^{m} t_j = 1$
A finite FC	$\mathbf{H}$
A finite FCAS	$(W, \mathbf{H})$

Table 3
The multi-criteria information table.

$W/\mathbf{H}$	H_1	H_2	$\cdots$	H_m
k_1	h_{11}	h_{12}	$\cdots$	h_{1m}
k_2	h_{21}	h_{22}	$\cdots$	h_{2m}
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
k_n	h_{n1}	h_{n2}	$\cdots$	h_{nm}
Weight (T)	t_1	t_2	$\cdots$	t_m

Table 4
The lower approximation information matrix.

$W/\mathbf{H}$	H_1	H_2	$\cdots$	H_m
k_1	u_{11}	u_{12}	$\cdots$	u_{1m}
k_2	u_{21}	u_{22}	$\cdots$	u_{2m}
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
k_n	u_{n1}	u_{n2}	$\cdots$	u_{nm}
k_1	u_{11}	u_{12}	$\cdots$	u_{1m}
k_2	u_{21}	u_{22}	$\cdots$	u_{2m}
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
k_n	u_{n1}	u_{n2}	$\cdots$	u_{nm}

Table 5
The upper approximation information matrix.

$W/\mathbf{H}$	H_1	H_2	$\cdots$	H_m
k_1	v_{11}	v_{12}	$\cdots$	v_{1m}
k_2	v_{21}	v_{22}	$\cdots$	v_{2m}
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
k_n	v_{n1}	v_{n2}	$\cdots$	v_{nm}

$$u_{j,-}(v_{j,-}) = \begin{cases} \min\limits_{1\leqslant i\leqslant n} u_{ij}\left(\min\limits_{1\leqslant i\leqslant n} v_{ij}\right), & H_j \in C, j \in \{1,2,\cdots,m\}, \\ \max\limits_{1\leqslant i\leqslant n} u_{ij}\left(\max\limits_{1\leqslant i\leqslant n} v_{ij}\right), & H_j \in D, j \in \{1,2,\cdots,m\}, \end{cases} \tag{4-5}$$

here C and D are the set of benefit and cost criteria, respectively.

The distance formula in the fuzzy information matrix is displayed as follows:

$$d(x,y) = |x-y|, x,y \in [0,1], \tag{4-6}$$

where $|\Box|$ represents the absolute value of $\Box$.

Therefore, we can obtain the lower approximation positive ideal distance matrix U_+ and the lower approximation negative ideal distance matrix U_- as follows:

$$U_+ = \begin{bmatrix} u_{11,+} & u_{12,+} & \cdots & u_{1m,+} \\ u_{21,+} & u_{22,+} & \cdots & u_{2m,+} \\ \vdots & \vdots & \cdots & \vdots \\ u_{n1,+} & u_{n2,+} & \cdots & u_{nm,+} \end{bmatrix}, \tag{4-7}$$

$$U_- = \begin{bmatrix} u_{11,-} & u_{12,-} & \cdots & u_{1m,-} \\ u_{21,-} & u_{22,-} & \cdots & u_{2m,-} \\ \vdots & \vdots & \cdots & \vdots \\ u_{n1,-} & u_{n2,-} & \cdots & u_{nm,-} \end{bmatrix}, \tag{4-8}$$

where $u_{ij,+} = d(u_{j,+} - u_{ij})$, $u_{ij,-} = d(u_{j,-} - u_{ij})$.

Likewise, the upper approximation positive ideal distance matrix V_+ and the upper approximation negative ideal distance matrix V_- are shown as follows:

$$V_+ = \begin{bmatrix} v_{11,+} & v_{12,+} & \cdots & v_{1m,+} \\ v_{21,+} & v_{22,+} & \cdots & v_{2m,+} \\ \vdots & \vdots & \cdots & \vdots \\ v_{n1,+} & v_{n2,+} & \cdots & v_{nm,+} \end{bmatrix}, \tag{4-9}$$

$$V_- = \begin{bmatrix} v_{11,-} & v_{12,-} & \cdots & v_{1m,-} \\ v_{21,-} & v_{22,-} & \cdots & v_{2m,-} \\ \vdots & \vdots & \cdots & \vdots \\ v_{n1,-} & v_{n2,-} & \cdots & v_{nm,-} \end{bmatrix}, \tag{4-10}$$

where $v_{ij,+} = d(v_{j,+} - v_{ij})$, $v_{ij,-} = d(v_{j,-} - v_{ij})$.

Through the matrices (4–7) and (4–8), we obtain the correlation coefficient value of alternative k_i on criterion H_j. These correlation coefficient values form the lower approximation correlation matrix U_* as follows:

$$U_* = \begin{bmatrix} u_{11,*} & u_{12,*} & \cdots & u_{1m,*} \\ u_{21,*} & u_{22,*} & \cdots & u_{2m,*} \\ \vdots & \vdots & \cdots & \vdots \\ u_{n1,*} & u_{n2,*} & \cdots & u_{nm,*} \end{bmatrix}, \tag{4-11}$$

where

$$u_{ij,*} = \frac{u_{ij,-}}{u_{ij,-} + u_{ij,+}}, i \in \{1, 2, \cdots, n\}, j \in \{1, 2, \cdots, m\}.$$

Likewise, through the matrices (4–9) and (4–10), the upper approximation correlation matrix V_* is listed as follows:

$$V_* = \begin{bmatrix} v_{11,*} & v_{12,*} & \cdots & v_{1m,*} \\ v_{21,*} & v_{22,*} & \cdots & v_{2m,*} \\ \vdots & \vdots & \cdots & \vdots \\ v_{n1,*} & v_{n2,*} & \cdots & v_{nm,*} \end{bmatrix}, \tag{4-12}$$

where

$$v_{ij,*} = \frac{v_{ij,-}}{v_{ij,-} + v_{ij,+}}, i \in \{1, 2, \cdots, n\}, j \in \{1, 2, \cdots, m\}.$$

By the lower approximation correlation matrix (4–11), each alternative has a correlation coefficient value on each criterion. For each alternative, we use the WAA operator to aggregate the correlation coefficient values on each criterion. Then, we can obtain an overall coefficient value. The specific way is listed as follows:

$$k_{i,u,*} = t_1 u_{i1,*} + t_2 u_{i2,*} + \cdots + t_m u_{im,*}, i \in \{1, 2, \cdots, n\}. \tag{4-13}$$

Likewise, by the upper approximation correlation matrix (4–12), the overall coefficient value of each alternative is listed as follows:

$$k_{i,v,*} = t_1 v_{i1,*} + t_2 v_{i2,*} + \cdots + t_m v_{im,*}, i \in \{1, 2, \cdots, n\}. \tag{4-14}$$

Using the two overall coefficient values formed by the lower approximation correlation matric and upper approximation correlation matric, we use the following method to obtain the intimacy coefficient of each alternative.

$$k_{i,*} = k_{i,u,*} + k_{i,v,*} - k_{i,u,*} k_{i,v,*}, i \in \{1, 2, \cdots, n\}. \tag{4-15}$$

We can acquire the ranking result of alternatives by the size of intimacy coefficient $k_{i,*}$. The larger $k_{i,*}$ is, the better alternative k_i is.

Remark 4.1. The following statements will explain the rationality and advantage of decision-making method.

(1) In this paper, we focus on how to deal with MCDM problems in a finite FCAS. In light of this, we combine the TOPSIS method, the WAA operator method with CFRS theory to propose the TOPSIS-WAA method in a finite FCAS. As we all know, both the TOPSIS method and the WAA operator method are effective methods for dealing with multi-criteria problems.

Furthermore, CFRS theory is an effective tool for dealing with inaccurate and ambiguous data in a finite FCAS. Our method not only inherits advantages of these three theories, but also effectively solves the multi-criteria problem in a finite FCAS.
(2) According to the above descriptions, we find that a multi-criteria information table can form a finite FCAS. However, in Example 3.1, the failure of the TOPSIS method and the WAA operator method in a finite FCAS brings us some questions that need to be solved urgently. In order to deal with the multi-criteria problem in a finite FCAS, we develop a new MCDM method. This method is an effective combination of the TOPSIS method, the WAA operator method and CFRS theory. The core ideas of our method are as follows: first, we utilize the possible rule and the deterministic rule in CFRS theory to describe the fuzzy sets formed by the criteria. Then, we obtain two new multi-criteria fuzzy information matrices. These two fuzzy information matrices form two finite FCASs. Secondly, based on these two fuzzy information matrices, we use the idea of the TOPSIS to obtain the lower approximation (deterministic) correlation matrix and the upper approximation (possible) correlation matrix. Then, for each alternative, we use the idea of the WAA operator to obtain the two overall coefficient values formed by lower and upper approximation correlation matrices. Finally, for each alternative, we utilize the idea of probability sum function to fuse these two overall coefficient values into one intimacy coefficient value. According to intimacy coefficient values of all alternatives, we can obtain an optimal ranking scheme.
(3) In a finite FCAS, the TOPSIS method and the WAA operator method may fail under the influence of some special subjective weights (Example 3.1). But our method can directly ignore the adverse effects of subjective weight.

4.3. Method steps

The detailed steps of our method are shown as follows:
Input The MCDM information.
Output The ranking for all alternatives.
Step 1 : According to the actual problem, we acquire an MCDM matrix with fuzzy information that can form a finite FCAS.
Step 2 : By the model $\left(\underline{F}_{N_1^{\mathbf{H}},\mathscr{T},\mathscr{I}},\overline{F}_{N_1^{\mathbf{H}},\mathscr{I},\mathscr{T}}\right)$, formulas (4–2) and (4–3), we obtain the lower and upper approximation information matrices with fuzzy information, respectively.
Step 3 : By the Table 4, Table 5 and the formulas (4–4)-(4–6), we obtain the lower (upper) approximation positive ideal distance matrix and the lower (upper) approximation negative ideal distance matrix.
Step 4 : By the matrices (4–7)-(4–10), we obtain the lower (upper) approximation correlation matrix U_* (V_*).
Step 5 : By the formulas (4–13)-(4–15), we obtain the ranking result of all alternatives.

Remark 4.2. The global algorithm complexity of our method is the sum of the local algorithm complexity of each step in the method. In fact, the local algorithm complexity of Step 2 is the highest, namely $O(mn^3)$. In Step 2, we need to calculate the lower and upper approximation sets of m fuzzy sets. In the calculation of the lower or upper approximation set of each fuzzy set, n^2 operations are required for each object. Then, the lower or upper approximation set of each fuzzy set requires n^3 operations. Thus, the number of operations in Step 2 is mn^3. Note that the number of operations in other steps is actually lower than the number of operations in Step 2. Therefore, the global algorithm complexity of our method is $O(mn^3)$.

To improve the visibility, the following Fig. 2 shows the overall idea of the method for this paper.

5. Problem background, scheme description, numerical simulation and validity test

In Section 4, we establish the TOPSIS-WAA method in a finite FCAS. In real life, there are some MCDM problems that can form finite FCASs. In this section, we use the decision method established in this paper to propose a rating scheme to apply the rating problem of some specific customer groups.

5.1. Problem background

Preferential activities are an important means for some merchants or companies to return new and regular customers. However, the resources in these activities are limited. In other words, the number of customers who can enjoy the preferential service is limited. Moreover, the level of these preferential services may also be different, that is, the preferential service can be divided into the first-level preferential service, the second-level preferential service, and so on. At the same time, the number of preferential services at each level is also different. Therefore, merchants or enterprises need to formulate a scheme to distribute these preferential resources to customers fairly. In fact, the essence of this scheme is the rating scheme, which divides all customers into multiple different levels.

5.2. Scheme description

Based on the above problem, we design a rating scheme based on an MCDM method in a finite FCAS in this paper. The rating scheme is implemented based on the preset subjective ratio and the calculated objective ranking of all alternatives.

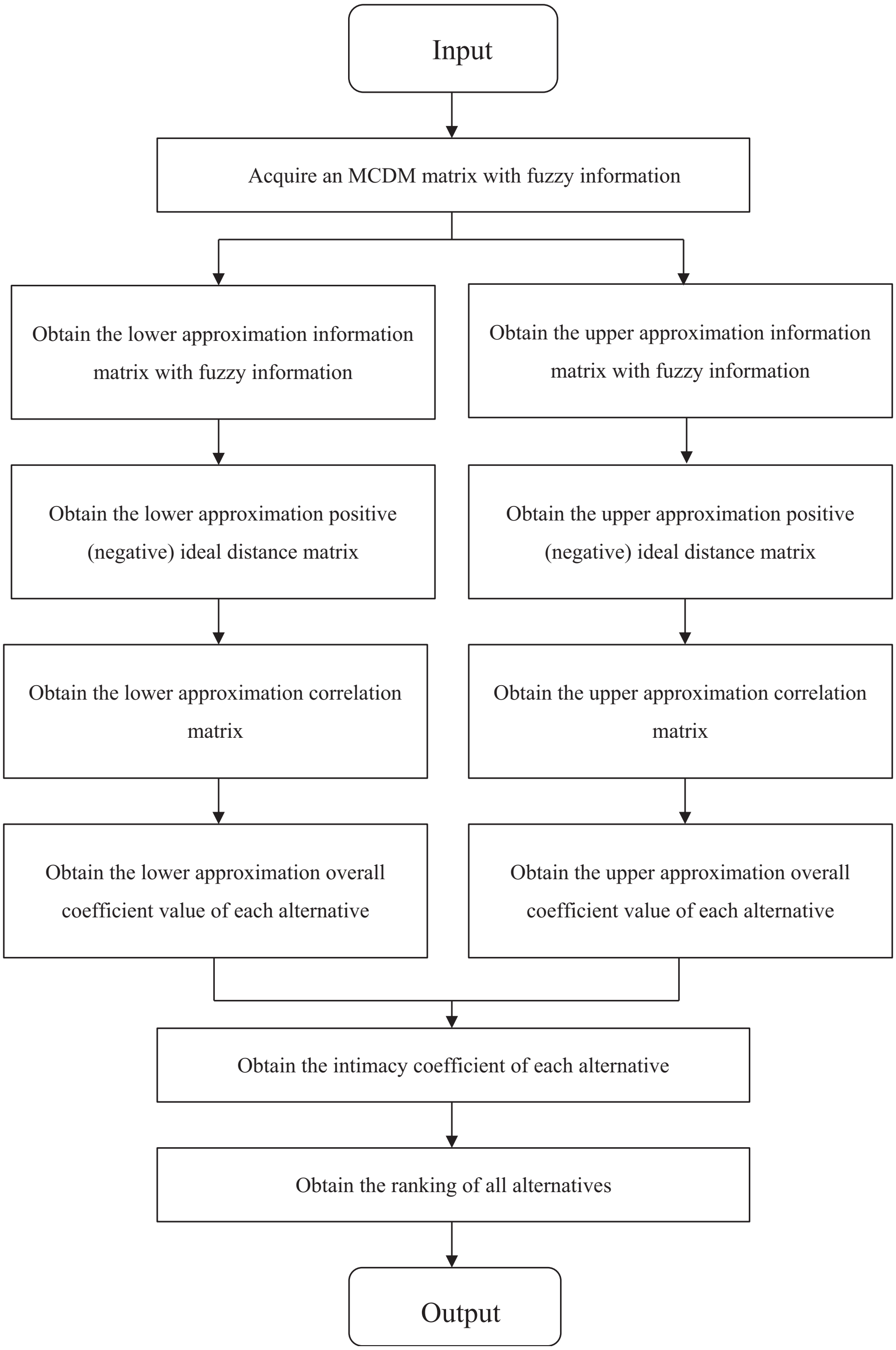

Fig. 2. The decision-making step.

That is to say, the scheme mainly includes two aspects, namely the quantity ratio among all levels and the ranking information among all customers.

For these two aspects, our explanations are as follows:

(1) The quantity ratio among all levels is actually very easy to be determined. This ratio can be determined according to the number of preferential services at various levels in actual activities. In other words, the ratio can be determined by the merchants (or decision-makers) in advance, which is called the subjective ratio.

(2) The ranking information among all customers can be obtained by the ranking decision method provided in this paper.

The detailed steps of the rating scheme are as follows:

Firstly, all customers are scored according to pre-determined criteria. Note that the score interval is [0, 1]. The score information of all customers forms a multi-criteria fuzzy information matrix or a finite FCAS.

Secondly, all customers' ranking information is obtained by the MCDM method provided in this paper. That is, merchants or companies can rank all customers. The higher the customer's ranking, the better the preferential service.

Finally, merchants or companies determine the number of customers in each level (i.e., the quantitative ratio) according to the actual situation. At the same time, based on the known ranking information of all customers, each customer is determined to be a certain level. Based on this, the levels of all customers are determined. Moreover, the higher the customer's ranking, the higher the rating, and the better the preferential service.

In addition, the customer's score information under each criterion is obtained according to the following principles:

- Based on each criterion, we first convert the customer's original information into a score with fuzzy information (the score interval is [0, 1]). Scores with fuzzy information are obtained by predetermined score conversion rules. The score conversion rules are set by merchants or companies (decision-makers).
- To prevent excessive gaps among customers, we set an upper limit for each criterion. If a customer's original information value exceeds this limit, then the score with fuzzy information of a customer is 1.
- We define the following principles. If a customer's scores are 1 on all criteria, he or she becomes a highest-level directly. If a customer's scores are no more than 1 on all criteria, he or she becomes a lowest-level directly. Therefore, these two types of customers are excluded directly when classifying all customers.

  According to the above principles, we can convert any data set into fuzzy information data. Based on the new fuzzy information data, we can obtain an MCDM data table (or information system) with fuzzy information. By Definition 2.1, we can see that this multi-criteria information system is a finite FCAS.

Next, we use the scheme proposed in this paper to solve the rating problem in real life.

5.3. Numerical simulation

This paper takes the bank's implementation of preferential business for some customers as an example. At the same time, we extract some of the data from the customer information database of ICBC in 2018 to conduct experiments.

In this paper, we fix the upper limit of each criterion. The purpose is to unify the dimensions on different criteria. At the same time, we propose the following conversion scheme to obtain customers' fuzzy evaluation scores on the criteria. This conversion scheme can be regarded as fuzzy granulation of the evaluation value on the criterion. If the customer's data value E on a certain criterion is higher than the upper limit F of criterion, it is directly recorded as 1. If the customer's data value E on a certain criterion is lower than the upper limit F of criterion, it is recorded as $\frac{E}{F}$.

Thus, for six criteria, our explanations are shown as follows:

Deposit situation (H_1): This is a benefit criterion. The more deposit amount, the higher score. Moreover, the upper limit of deposit situation is 300,000 RMB. The fuzzy evaluation value $H_1(k)$ obtained by the object k on the criterion H_1 can be obtained by the following formula:

$$H_1(k) = \begin{cases} \frac{E}{300000}, & E \in [0, 300000), \\ 1, & E \in [300000, \infty). \end{cases} \tag{4-16}$$

Personal monthly income (H_2): This is a benefit criterion. Moreover, the upper limit of personal monthly income is 10,000 RMB. The fuzzy evaluation value $H_2(k)$ obtained by the object k on the criterion H_2 can be obtained by the following formula:

$$H_2(k) = \begin{cases} \frac{E}{10000}, & E \in [0, 10000), \\ 1, & E \in [10000, \infty). \end{cases} \tag{4-17}$$

Degree of education (H_3): This is a benefit criterion. In general, score interval of this criterion is $[0, 10]$. Moreover, the upper limit of this criterion is 6. The fuzzy evaluation value $H_3(k)$ obtained by the object k on the criterion H_3 can be obtained by the following formula:

$$H_3(k) = \begin{cases} \frac{E}{6}, & E \in [0, 6), \\ 1, & E \in [6, 10]. \end{cases} \tag{4-18}$$

Career situation (H_4): This is a benefit criterion. Bank staffs will predetermine relevant scores for different occupations. In general, score interval of this criterion is $[0, 10]$. Moreover, the upper limit of the score of this criterion is 6. The fuzzy evaluation value $H_4(k)$ obtained by the object k on the criterion H_4 can be obtained by the following formula:

$$H_4(k) = \begin{cases} \frac{E}{6}, & E \in [0, 6), \\ 1, & E \in [6, 10]. \end{cases} \tag{4-19}$$

Debt situation (H_5): This is obviously a cost criterion. The upper limit of the debt situation is 50,000 RMB. In addition, for the fuzzy information of customer under cost criterion, we use the difference value method to convert it into fuzzy information under the benefit criterion. The principle of difference value method is as follows: If the cost fuzzy information is θ ($\theta \in [0,1]$), then the benefit fuzzy information is $1-\theta$. This way is used to facilitate our unified calculation. The fuzzy evaluation value $H_5(k)$ obtained by the object k on the criterion H_5 can be obtained by the following formula:

$$H_5(k) = \begin{cases} 1 - \frac{E}{50000}, & E \in [0, 50000), \\ 0, & E \in [50000, \infty). \end{cases} \tag{4-20}$$

Interview situation (H_6): This is a benefit criterion. When a bank staff collects information from customers, the performance of customers in the process of communicating with the bank staff also affects customer's credit rating. In general, score interval of this criterion is $[0,10]$. Moreover, the upper limit of score of this criterion is 6. The fuzzy evaluation value $H_6(k)$ obtained by the object k on the criterion H_6 can be obtained by the following formula:

$$H_6(k) = \begin{cases} \frac{E}{6}, & E \in [0, 6), \\ 1, & E \in [6, 10]. \end{cases} \tag{4-21}$$

Based on each criterion, the process of obtaining fuzzy evaluation values is the process of implementing fuzzy granulation. By the formulas (4–16)-(4–21), we can convert the original evaluation values obtained by different objects on different criteria into fuzzy evaluation values.

In order to facilitate the calculation, we extract 10 customers as a sample of the data experiments in this paper. Moreover, we mark CU_{1548}, CU_{2859}, CU_{6984}, CU_{12587}, CU_{18955}, CU_{25896}, CU_{65897}, CU_{78521}, CU_{87457}, CU_{98512} as k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}, respectively. **At the same time, in this paper, we divide the preferential business into four levels (A, B, C, D), and the subjective ratio is 1: 2: 3: 4.** Data are as follows:

Before the formal calculation, we convert the actual data (Table 7) into the fuzzy information data (Table 8) according to the formulas (4–16)-(4–21). For example, the data value of the customer CU_{1548} on H_1 is 300200 in Table 6. The upper limit of the criterion H_1 is 300,000. Therefore, the fuzzy information value of the customer k_1 on H_1 is 1 in Table 8. In addition, the data value of the customer CU_{1548} on H_2 is 6400 in Table 6. The upper limit of the criterion H_2 is 10,000. Therefore, the fuzzy information value of the customer k_1 on H_2 is $\frac{6400}{10000} = 0.64$ in Table 8.

Based on the above descriptions, the fuzzy information data of 10 customers are shown in Table 8.

Step 1 : The fuzzy information matrix is shown in Table 8.

Step 2 : For every $H \in \mathscr{F}(W)$ and $e \in W$, the specific form of $\left(\underline{F}_{N_1^{\mathbf{H}},\mathscr{T},\mathscr{I}}(H), \overline{F}_{N_1^{\mathbf{H}},\mathscr{I},\mathscr{T}}(H)\right)$ is listed as follows:

$$\underline{F}_{N_1^{\mathbf{H}},\mathscr{T},\mathscr{I}}(H)(e) = \bigvee_{f \in W} \mathscr{T}\left(N_1^{\mathbf{H}}(f)(e), \bigwedge_{g \in W} \mathscr{I}\left(N_1^{\mathbf{H}}(f)(g), H(g)\right)\right), \tag{4-22}$$

$$\overline{F}_{N_1^{\mathbf{H}},\mathscr{I},\mathscr{T}}(H)(e) = \bigwedge_{f \in W} \mathscr{I}\left(N_1^{\mathbf{H}}(f)(e), \bigvee_{g \in W} \mathscr{T}\left(N_1^{\mathbf{H}}(f)(g), H(g)\right)\right). \tag{4-23}$$

Table 6
The customer information data of ICBC in 2018.

$W/\mathbf{H}$	H_1	H_2	H_3	H_4	H_5	H_6
...	...	...	...	...	...	...
CU_{1548}	300200	6400	1.5	5.58	29000	3.54
...	...	...	...	...	...	...
CU_{2859}	168000	7600	1.74	5.82	50320	3.9
...	...	...	...	...	...	...
CU_{6984}	264000	10000	4.26	4.68	20500	4.56
...	...	...	...	...	...	...
CU_{12587}	198000	8300	7	4.02	26500	4.92
...	...	...	...	...	...	...
CU_{18955}	300000	4800	4.92	3.24	15500	5.04
...	...	...	...	...	...	...
CU_{25896}	117000	2600	5.7	6	500	5.76
...	...	...	...	...	...	...
CU_{65897}	138000	9500	4.38	7.2	7500	3.18
...	...	...	...	...	...	...
CU_{78521}	294000	6200	6	2.22	18000	3.06
...	...	...	...	...	...	...
CU_{87457}	300800	4600	3.48	0.96	6000	3.36
...	...	...	...	...	...	...
CU_{98512}	255000	5800	6	5.28	2000	3.66
...	...	...	...	...	...	...

Table 7
The MCDM matrix.

$W/\mathbf{H}$	H_1	H_2	H_3	H_4	H_5	H_6
k_1	300200	6400	1.5	5.58	29000	3.54
k_2	168000	7600	1.74	5.82	50320	3.9
k_3	264000	10000	4.26	4.68	20500	4.56
k_4	198000	8300	7	4.02	26500	4.92
k_5	300000	4800	4.92	3.24	15500	5.04
k_6	117000	2600	5.7	6	500	5.76
k_7	138000	9500	4.38	7.2	7500	3.18
k_8	294000	6200	6	2.22	18000	3.06
k_9	300800	4600	3.48	0.96	6000	3.36
k_{10}	255000	5800	6	5.28	2000	3.66

Table 8
The fuzzy information matrix.

$W/\mathbf{H}$	H_1	H_2	H_3	H_4	H_5	H_6
k_1	1	0.64	0.25	0.93	0.42	0.59
k_2	0.56	0.76	0.29	0.97	1	0.65
k_3	0.88	1	0.71	0.78	0.59	0.76
k_4	0.66	0.83	1	0.67	0.47	0.82
k_5	1	0.48	0.82	0.54	0.69	0.84
k_6	0.39	0.26	0.98	1	0.99	0.96
k_7	0.46	0.95	0.73	1	0.85	0.53
k_8	0.98	0.62	1	0.37	0.64	0.51
k_9	1	0.46	0.58	0.16	0.88	0.56
k_{10}	0.85	0.58	1	0.88	0.96	0.61
T	0.25	0.25	0.15	0.15	0.1	0.1

Table 9
The lower approximation fuzzy information matrix.

$W/\mathbf{H}$	H_1	H_2	H_3	H_4	H_5	H_6
k_1	1	0.6400	0.2500	0.9300	0.4200	0.5900
k_2	0.5600	0.7600	0.2900	0.9700	1	0.6500
k_3	0.8800	1	0.7100	0.7800	0.5900	0.7600
k_4	0.6600	0.8300	1	0.6700	0.4700	0.8200
k_5	1	0.4800	0.8200	0.5400	0.6900	0.8400
k_6	0.3900	0.2600	0.9800	1	0.9900	0.9600
k_7	0.4600	0.9500	0.7300	1	0.8500	0.5300
k_8	0.9800	0.6200	1	0.3700	0.6400	0.5100
k_9	1	0.4600	0.5800	0.1600	0.8800	0.5600
k_{10}	0.8500	0.5800	1	0.8800	0.9600	0.6100
k_1	1	0.6400	0.2500	0.9300	0.4200	0.5900
k_2	0.5600	0.7600	0.2900	0.9700	1	0.6500
k_3	0.8800	1	0.7100	0.7800	0.5900	0.7600
k_4	0.6600	0.8300	1	0.6700	0.4700	0.8200
k_5	1	0.4800	0.8200	0.5400	0.6900	0.8400
k_6	0.3900	0.2600	0.9800	1	0.9900	0.9600
k_7	0.4600	0.9500	0.7300	1	0.8500	0.5300
k_8	0.9800	0.6200	1	0.3700	0.6400	0.5100
k_9	1	0.4600	0.5800	0.1600	0.8800	0.5600
k_{10}	0.8500	0.5800	1	0.8800	0.9600	0.6100

Let $\mathscr{T} = \mathscr{T}_p$[3], $\mathscr{I} = \mathscr{I}_p$[4], then the lower and upper approximation fuzzy information matrices are shown in Table 9 and Table 10, respectively.

Remark 5.1. In fact, we can also use other fuzzy logical operator order pairs ($(\mathscr{T}_L, \mathscr{I}_L)$ and $(\mathscr{T}_M, \mathscr{I}_M)$) to calculate the lower and upper approximations of H.

[3] $\mathscr{T}_p(b,c) = b * c$ for every $b, c \in [0,1]$.
[4] $\mathscr{I}_P(b,c) = \frac{c}{b}$ for $b > c$ and $\mathscr{I}_P(b,c) = 1$ elsewhere, for any $e,f \in [0,1]$.

Table 10
The upper approximation fuzzy information matrix.

$W/\mathbf{H}$	H_1	H_2	H_3	H_4	H_5	H_6
k_1	1	0.8387	0.8500	0.9300	0.8160	0.6374
k_2	0.6487	0.7746	0.7632	0.9700	1	0.6500
k_3	0.8800	1	0.7500	0.7800	0.5900	0.7600
k_4	0.6600	0.8300	1	0.6700	0.6708	0.8200
k_5	1	0.8551	0.8200	0.6670	0.6971	0.8400
k_6	0.5401	0.5960	0.9800	1	0.9900	0.9600
k_7	0.6108	0.9500	0.7300	1	0.8500	0.5300
k_8	0.9800	0.7100	1	0.7633	0.8327	0.6503
k_9	1	0.6705	0.8500	0.7480	0.8800	0.6586
k_{10}	0.8500	0.6146	1	0.8800	0.9600	0.6100

Table 11
The lower approximation positive (negative) ideal point $u_{j,+}$ ($u_{j,-}$).

	H_1	H_2	H_3	H_4	H_5	H_6
$u_{j,+}$	1	1	1	1	1	0.9600
$u_{j,-}$	0.3900	0.2600	0.2500	0.1600	0.4200	0.5100

Table 12
The upper approximation positive (negative) ideal point $v_{j,+}$ ($v_{j,-}$).

	H_1	H_2	H_3	H_4	H_5	H_6
$v_{j,+}$	1	1	1	1	1	0.9600
$v_{j,-}$	0.5401	0.5960	0.7300	0.6670	0.5900	0.5300

Step 3 : According to Table 9, the formula (4–4) and the formula (4–5), the lower approximation positive and negative ideal points on criterion H_j are listed as follows:

Likewise, by Table 10, the upper approximation positive and negative ideal points on criterion H_j are listed as follows: see Tables 11 and 12.

Thus, the lower approximation positive ideal distance matrix U_+ and the lower approximation negative ideal distance matrix U_- are listed as follows:

$$U_+ = \begin{bmatrix} 0 & 0.3600 & 0.7500 & 0.0700 & 0.5800 & 0.3700 \\ 0.4400 & 0.2400 & 0.7100 & 0.0300 & 0 & 0.3100 \\ 0.1200 & 0 & 0.2900 & 0.2200 & 0.41 & 0.2000 \\ 0.3400 & 0.1700 & 0 & 0.3300 & 0.5300 & 0.1400 \\ 0 & 0.5200 & 0.1800 & 0.4600 & 0.3100 & 0.1200 \\ 0.6100 & 0.7400 & 0.0200 & 0 & 0.0100 & 0 \\ 0.5400 & 0.0500 & 0.2700 & 0 & 0.1500 & 0.4300 \\ 0.0200 & 0.3800 & 0 & 0.6300 & 0.3600 & 0.4500 \\ 0 & 0.5400 & 0.4200 & 0.8400 & 0.1200 & 0.4000 \\ 0.1500 & 0.4200 & 0 & 0.1200 & 0.0400 & 0.3500 \end{bmatrix}, \tag{4-24}$$

$$U_- = \begin{bmatrix} 0.6100 & 0.3800 & 0 & 0.7700 & 0 & 0.0800 \\ 0.1700 & 0.5000 & 0.0400 & 0.8100 & 0.5800 & 0.1400 \\ 0.4900 & 0.7400 & 0.4600 & 0.6200 & 0.1700 & 0.2500 \\ 0.2700 & 0.5700 & 0.7500 & 0.5100 & 0.0500 & 0.3100 \\ 0.6100 & 0.2200 & 0.5700 & 0.3800 & 0.2700 & 0.3300 \\ 0 & 0 & 0.7300 & 0.8400 & 0.5700 & 0.4500 \\ 0.0700 & 0.6900 & 0.4800 & 0.8400 & 0.4300 & 0.0200 \\ 0.5900 & 0.3600 & 0.7500 & 0.2100 & 0.2200 & 0 \\ 0.6100 & 0.2000 & 0.3300 & 0 & 0.4600 & 0.0500 \\ 0.4600 & 0.3200 & 0.7500 & 0.7200 & 0.5400 & 0.1000 \end{bmatrix}. \tag{4-25}$$

Likewise, the upper approximation positive ideal distance matrix V_+ and the upper approximation negative ideal distance matrix V_- are listed as follows:

$$V_+ = \begin{bmatrix} 0 & 0.1613 & 0.1500 & 0.0700 & 0.1840 & 0.3226 \\ 0.3513 & 0.2254 & 0.2368 & 0.0300 & 0 & 0.3100 \\ 0.1200 & 0 & 0.2500 & 0.2200 & 0.4100 & 0.2000 \\ 0.3400 & 0.1700 & 0 & 0.3300 & 0.3292 & 0.1400 \\ 0 & 0.1499 & 0.1800 & 0.3330 & 0.3029 & 0.1200 \\ 0.4599 & 0.4040 & 0.0200 & 0 & 0.0100 & 0 \\ 0.3892 & 0.0500 & 0.2700 & 0 & 0.1500 & 0.4300 \\ 0.0200 & 0.2900 & 0 & 0.2367 & 0.1673 & 0.3097 \\ 0 & 0.3295 & 0.1500 & 0.2520 & 0.1200 & 0.3014 \\ 0.1500 & 0.3854 & 0 & 0.1200 & 0.0400 & 0.3500 \end{bmatrix}, \tag{4-26}$$

$$V_- = \begin{bmatrix} 0.4599 & 0.2427 & 0.1200 & 0.2630 & 0.2260 & 0.1074 \\ 0.1086 & 0.1786 & 0.0332 & 0.3030 & 0.4100 & 0.1200 \\ 0.3399 & 0.4040 & 0.0200 & 0.1130 & 0 & 0.2300 \\ 0.1199 & 0.2340 & 0.2700 & 0.0030 & 0.0808 & 0.2900 \\ 0.4599 & 0.2591 & 0.0900 & 0 & 0.1071 & 0.3100 \\ 0 & 0 & 0.2500 & 0.3330 & 0.4000 & 0.4300 \\ 0.0707 & 0.3540 & 0 & 0.3330 & 0.2600 & 0 \\ 0.4399 & 0.1140 & 0.2700 & 0.0963 & 0.2427 & 0.1203 \\ 0.4599 & 0.0745 & 0.1200 & 0.0810 & 0.2900 & 0.1286 \\ 0.3099 & 0.0186 & 0.2700 & 0.2130 & 0.3700 & 0.0800 \end{bmatrix}. \tag{4-27}$$

Step 4 : By the matrices (4–24) and (4–25), the lower approximation correlation matrix U_* is listed as follows:

$$U_* = \begin{bmatrix} 1 & 0.5135 & 0 & 0.9167 & 0 & 0.1778 \\ 0.2787 & 0.6757 & 0.0533 & 0.9643 & 1 & 0.3111 \\ 0.8033 & 1 & 0.6133 & 0.7381 & 0.2931 & 0.5556 \\ 0.4426 & 0.7703 & 1 & 0.6071 & 0.0862 & 0.6889 \\ 1 & 0.2973 & 0.7600 & 0.4524 & 0.4655 & 0.7333 \\ 0 & 0 & 0.9733 & 1 & 0.9828 & 1 \\ 0.1148 & 0.9324 & 0.6400 & 1 & 0.7414 & 0.0444 \\ 0.9672 & 0.4865 & 1 & 0.2500 & 0.3793 & 0 \\ 1 & 0.2703 & 0.4400 & 0 & 0.7931 & 0.1111 \\ 0.7541 & 0.4324 & 1 & 0.8571 & 0.9310 & 0.2222 \end{bmatrix}. \tag{4-28}$$

By the matrices (4–26) and (4–27), the upper approximation correlation matrix V_* is listed as follows:

$$V_* = \begin{bmatrix} 1 & 0.6007 & 0.4444 & 0.7898 & 0.5512 & 0.2498 \\ 0.2361 & 0.4421 & 0.1230 & 0.9099 & 1 & 0.2791 \\ 0.7391 & 1 & 0.0741 & 0.3393 & 0 & 0.5349 \\ 0.2607 & 0.5792 & 1 & 0.0090 & 0.1971 & 0.6744 \\ 1 & 0.6413 & 0.3333 & 0 & 0.2612 & 0.7209 \\ 0 & 0 & 0.9259 & 1 & 0.9756 & 1 \\ 0.1537 & 0.8762 & 0 & 1 & 0.6341 & 0 \\ 0.9565 & 0.2822 & 1 & 0.2892 & 0.5920 & 0.2798 \\ 1 & 0.1844 & 0.4444 & 0.2432 & 0.7073 & 0.2991 \\ 0.6738 & 0.0460 & 1 & 0.6396 & 0.9024 & 0.1860 \end{bmatrix}. \tag{4-29}$$

Step 5 : By the matrices (4–28) and (4–29), the two overall coefficient values are listed as follows:
Finally, by Table 13, the intimacy coefficients of all alternatives are shown as follows:
By Table 14, the ranking result of all alternatives is listed as follows:

$$k_3 \succ k_{10} \succ k_1 \succ k_5 \succ k_8 \succ k_4 \succ k_7 \succ k_6 \succ k_2 \succ k_9.$$

In this paper, we divide the preferential service into four levels (A, B, C, D) and set the ratio to 1: 2: 3: 4. Thus, for these ten customers, we have the following rating table:

Our rating principle and preferential principle are as follows: the higher the customer's ranking, the higher the rating, and the better the preferential service. Thus, customer k_3 enjoys the highest level (i.e., A) of preferential service, customers k_7, k_6, k_2, k_9 enjoy the lowest level (i.e., D) of preferential service.

Furthermore, customers of different levels can enjoy different preferential services, that is, the higher the level, the better the preferential service. We take k_1 and k_9 as examples. From Table 15, we find that k_1 is rated *B*-level and k_9 is rated *D*-level. Therefore, the degree of preferential service enjoyed by k_1 is higher than the degree of preferential service enjoyed by k_9.

Remark 5.2. Decision-makers can determine the number of preferential service levels and the quantitative ratio (subjective ratio) between each level according to the actual situation. **For example, the decision-maker can also divide the preferential service into three levels (High, Medium, Low), and the quantity ratio is 3: 3: 4.** Then, the new rating table is as follows: All in all, no matter how decision-makers determine the number of preferential service levels and the quantitative ratio (subjective ratio) among all levels, they need to follow a principle, namely, **the higher the customer's ranking, the higher the customer's rating, and the larger the preferential service.**

5.4. Validity of the method proposed in this paper

In the rating scheme, the ranking decision method proposed in this paper is the core. In order to further demonstrate the validity of our ranking method, we do the following two tests. The following test principle comes from [14,25,37].

(1) The first test rule: An effective decision-making method should meet the following requirement, that is, when the non-optimal alternative is replaced by another worse alternative, the optimal alternative is unchanged.
(2) The second test rule: If an original MCDM problem is decomposed into several small decision-making problems, the same decision-making method is used to obtain the rankings of these small decision-making problems. The ranking results of all alternatives obtained by several small problems should be consistent with the original ranking results obtained by the original MCDM problem. The decision method that satisfies the above test rule is called the effective decision method.

According to the above two test rules, our tests are as follows:

(1) **The first test**
Based on the numerical examples (Table 7) in Section 5.3, the optimal alternative calculated by our method is k_3, and the non-optimal alternative is k_9. At the same time, the rating table obtained from our rating scheme is shown in Table 15. In Table 7, we replace k_9 as 300000, 4000, 3, 0.6, 10000, 3. The new MCDM table (Table 17) after replacement is as follows: Using our method (all steps in Section 5.3), the sorting results are as follows:

$$k_3 \succ k_{10} \succ k_1 \succ k_5 \succ k_8 \succ k_4 \succ k_7 \succ k_6 \succ k_2 \succ k_9.$$

Based on the above ranking results, we find that the optimal alternative is still k_3. In addition, when the decision-maker sets the level of preferential service to four levels and the ratio is 1: 2: 3: 4, the rating table of all alternatives is still shown in Table 15. When the decision-maker sets the level of preferential service to three levels and the ratio is 3: 3: 4, the ranking table of all alternatives is still shown in Table 16. Thus, our method passes the first test.

(2) **The second test**
Based on the requirements of the second test, we decompose the original decision-making problem (the ranking problem of $\{k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}\}$ in Table 7) into two smaller decision problems, namely the ranking problems of $\{k_1, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}\}$ and $\{k_1, k_2, k_3, k_4, k_6, k_7, k_8, k_9, k_{10}\}$. Using our method (all steps in Section 5.3), the ranking result of $\{k_1, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}\}$ is $k_3 \succ k_{10} \succ k_1 \succ k_5 \succ k_8 \succ k_4 \succ k_7 \succ k_6 \succ k_9$, the ranking result of $\{k_1, k_2, k_3, k_4, k_6, k_7, k_8, k_9, k_{10}\}$ is $k_3 \succ k_{10} \succ k_1 \succ k_8 \succ k_4 \succ k_7 \succ k_6 \succ k_2 \succ k_9$. If the sorting results obtained by the two small problems are combined, then the final sorting result is $k_3 \succ k_{10} \succ k_1 \succ k_5 \succ k_8 \succ k_4 \succ k_7 \succ k_6 \succ k_2 \succ k_9$, which is consistent with the ranking results obtained from the original decision problem. Therefore, our method passes the second test.

In conclusion, our method is valid based on these two tests (the validation mechanism in [37]). Therefore, our method is valid.

6. Comparative analysis

In this section, we use several existing methods to compare our method and illustrate the advantages of our method.

Table 13
The overall coefficient values of all alternatives.

	k_1	k_2	k_3	k_4	k_5
$k_{i,u,*}$	0.5337	0.5223	0.7384	0.6218	0.6261
$k_{i,v,*}$	0.6654	0.4524	0.5503	0.4485	0.5585
	k_6	k_7	k_8	k_9	k_{10}
$k_{i,u,*}$	0.4943	0.5864	0.5889	0.4740	0.6905
$k_{i,v,*}$	0.4864	0.4709	0.5902	0.4999	0.5348

Table 14
The intimacy coefficients of all alternatives.

	k_1	k_2	k_3	k_4	k_5
$k_{i,*}$	0.8440	0.7384	0.8824	0.7914	0.8349
	k_6	k_7	k_8	k_9	k_{10}
$k_{i,*}$	0.7403	0.7812	0.8315	0.7369	0.8560

Table 15
The rating table.

Customers	Rating position
k_3	A
k_{10}, k_1	B
k_5, k_8, k_4	C
k_7, k_6, k_2, k_9	D

6.1. Comparison among different decision-making methods

The core of the rating scheme designed in this paper is an MCDM method based on a finite FCAS. In other words, the core of this rating problem is the ranking problem in a finite FCAS. Therefore, some existing methods [21,22,36,40] that using fuzzy rough set theory cannot solve such problems. For example, the method proposed by Zhan et al. [36] is suitable for solving multi-expert group decision-making problems; the method proposed by Zhang et al. [40] is to solve decision problems with intuitionistic fuzzy information.

At the same time, in order to verify the performance of our method, in this section, we will compare with our method with the TOPSIS method [12], the WAA operator method [28], the TOPSIS method based on a fuzzy β-covering approximation space [38], the TOPSIS method based on a variable-precision fuzzy β-covering approximation space [13] and the TOPSIS method based on a λ-rough set [33]. The core ideas of these MCDM methods are listed as follows:

- The TOPSIS method: The core idea of this method is to find the optimal solution that is close to the positive ideal solution but far from the negative ideal solution.
- The WAA operator method: The core of this method is a weighted arithmetic average operator. The basic idea of WAA is to use the weighted data to obtain the final value of each alternative.
- The TOPSIS method based on a fuzzy β-covering approximation space (TOPSISFβCAS): The core of this method is to utilize the relationships among the positive ideal fuzzy set, the negative ideal fuzzy set and the integrated ideal fuzzy set to obtain the intimacy coefficient of each alternative.
- The TOPSIS method based on a variable-precision fuzzy β-covering approximation space (the TOPSISVFCAS): The core of the method is to calculate the intimacy coefficient by using the positive ideal distance fuzzy set and the negative ideal distance fuzzy set.
- The TOPSIS method based on a λ-rough set (TOPSISλAS): The core of the method is to obtain the lower and upper approximation decision-making information system by means of λ-rough sets, and use the idea of the TOPSIS method to obtain the final order.

The reasons for comparing these five MCDM methods with ours are as follows:

(1) The TOPSIS method and the WAA operator method are both traditional decision methods. Both of them are suitable for solving decision problems with real-valued data. In essence, a finite FCAS can be regarded a system with real-valued information on [0,1]. Therefore, these two methods are also suitable for decision problems in finite FCASs. In view of this, this paper uses our method to compare with these two classical decision-making methods to verify the effectiveness of our method.

Table 16
The new rating table.

Customers	*rating position*
k_3, k_{10}, k_1	*High*
k_5, k_8, k_4	*Medium*
k_7, k_6, k_2, k_9	*Low*

Table 17
The new MCDM matrix.

$W/\mathbf{H}$	H_1	H_2	H_3	H_4	H_5	H_6
k_1	300200	6400	1.5	5.58	29000	3.54
k_2	168000	7600	1.74	5.82	50320	3.9
k_3	264000	10000	4.26	4.68	20500	4.56
k_4	198000	8300	7	4.02	26500	4.92
k_5	300000	4800	4.92	3.24	15500	5.04
k_6	117000	2600	5.7	6	500	5.76
k_7	138000	9500	4.38	7.2	7500	3.18
k_8	294000	6200	6	2.22	18000	3.06
k_9	300000	4000	3	0.6	10000	3
k_{10}	255000	5800	6	5.28	2000	3.66

(2) Both the TOPSISFβCAS and the TOPSISVFCAS are up-to-date methods and can solve the decision problem in the fuzzy β-covering approximation space. At the same time, when $\beta = 1$, a fuzzy β-covering approximation space is a finite FCAS. In view of this, this paper uses our method to compare with these two emerging methods to demonstrate the feasibility of our method.
(3) By setting the threshold, a finite FCAS can be transformed into a λ-approximation space. For this reason, this paper compares it with our method. Besides, the TOPSIS method based on a λ-rough set has the obvious drawback that the use of the deflection values is unreasonable when the final sort is calculated. We can witness this defect from the following comparative analysis.

In addition, note that any sorting decision-making methods are developed to obtain an optimal ranking scheme and an optimal alternative. A comparison among the above decision-making methods and our method will be reflected in these two aims. According to Table 8, we have the following results.

Fig. 3 shows the comparison between the TOPSIS method and our method, the comparison between the WAA operator method and our method, the comparison between the TOPSISFβCAS and our method, the comparison between the TOPSISVFCAS method and our method and the comparison between the TOPSISλAS and our method.

From Table 18 and Fig. 3, the description of ranking results of these six decision-making methods is as follows:

(1) Result description
All six decision-making methods have their own optimal ranking scheme. Furthermore, in addition to the TOPSISλAS, we can easily see that the optimal alternatives for other five decision-making methods are the same, namely k_3. In addition, we can find that the TOPSIS method, the WAA operator method and our method have the same second place, namely k_{10}. However, from the ranking results of the TOPSISFβCAS (β=0.6, α=0.7), we perceive that the alternative k_{10} is ranked 8th.
(2) Result analysis
Although these six methods all have a complete sorting scheme, their specific ordering is different. On the one hand, our method is highly similar to the WAA operator method in sorting results, and has a certain degree of similarity with the TOPSIS method. This phenomenon is reasonable. Although our method is a fusion and improvement of these two methods, it will not be exactly the same. This phenomenon indirectly explains the rationality and enforceability of our method. On the other hand, our method, the TOPSISFβCAS, the TOPSISVFCAS ($\alpha = 0, \beta = 0.6$) and the TOPSISλAS are both generalizations of the TOPSIS method in an FβCAS[5]. The best alternative for the first three methods is the same, i.e., k_3. This shows that our method is consistent with the latest methods [13,38]. However, we can see from Fig. 3 that the TOPSISλAS ($\lambda = 0.3$) not only fails to obtain a complete ordering, but also has different best alternative to other methods. This phenomenon does not meet the basic requirements of the ranking decision-making method.

Based on the above analysis and description, we notice that our method can be consistent with the traditional methods [12,28], and can have similarities with the latest methods [13,38]. This is a good indication of the rationality and enforceability of our method.

[5] When β=1, a fuzzy β-covering approximation space is a fuzzy covering approximation space.

6.2. Advantages of the method presented in this paper

In this section, we continue to illustrate the superiority of our method.

Notation : Given a collection of alternatives $W = \{k_1, k_2, \cdots, k_n\}$, based on a ranking decision-making method, we acquire an optimal ranking scheme $N : k_{i_1} \succ \cdots \succ k_{i_a} \succ \cdots \succ k_{i_b} \succ \cdots \succ k_{i_n}$, here $k_{i_1}, \cdots, k_{i_a}, \cdots, k_{i_b}, \cdots, k_{i_n}$ is an n-level permutation. For any two alternatives k_{i_a} and k_{i_b}, the relation $k_{i_a} \succ k_{i_b}$ is denoted an ordered relation. Thus, we can observe that the number of ordered relations of N is $(n-1) + (n-2) + \cdots + 2 + 1 = \frac{n(n-1)}{2}$. Based on the collection of alternatives $W = \{k_1, k_2, \cdots, k_n\}$, the sorting similarity degree between the two ranking schemes is explained as follows:

Consider two sorting schemes P, Q obtained by two different ranking decision-making methods. Based on W, the number of ordered relations in P and Q is both x. If there are y $(y \leqslant x)$ same ordered relations in P and Q, then the sorting similarity degree between P and Q is $\frac{y}{x}$. The sorting similarity degree represents the degree to which two ranking decision-making methods are consistent.

Remark 6.1.

(1) As mentioned above, our method and the TOPSISFβCAS are both generalizations of the TOPSIS method in an FβCAS. Therefore, based on the same data set, the ranking results of our method and the TOPSISFβCAS should be similar to the TOPSIS method. With notation and Table 18, we can calculate the degrees of consistency of our method and the TOPSIS method, the TOPSISFβCAS and the TOPSIS method are $\frac{34}{45}, \frac{28}{45}$, respectively. Therefore, compared to the TOPSISFβCAS, the consistency between our method and the TOPSIS method is higher. In other words, our method is better than the TOPSISFβCAS.

(2) At the same time, although the TOPSISVFCAS [13] is applicable to our rating scheme, it has some defects, that is, the model used in the article does not satisfy the inclusion relationship between the lower and upper approximations. This flaw can lead to unpredictability in lower and upper approximation calculations. Therefore, our method is better than the TOPSISVFCAS.

In addition, we can also illustrate that our method is superior to the WAA operator method and the TOPSIS method in some special situations.

Remark 6.2. In Example 3.1, we find that the WAA operator method and the TOPSIS method fail in a finite FCAS. In other words, the sorting result of both methods is $k_1 \approx k_2 \approx k_3 \approx k_4 \approx k_5 \approx k_6 \approx k_7 \approx k_8 \approx k_9 \approx k_{10}$. At the same time, we use our method to obtain the sorting result as $k_6 \succ k_9 \succ k_4 \succ k_7 \succ k_1 \succ k_5 \succ k_8 \succ k_2 \succ k_{10} \succ k_3$. This shows that our method has better sorting ability in complex fuzzy environment than the WAA operator method and the TOPSIS method.

In summary, we know that our method is not only consistent with the TOPSIS method (traditional method), the WAA operator method (traditional method) and the TOPSISFβCAS (latest method), but also has better sorting ability in complex fuzzy environment than these three methods.

Remark 6.3. In the light of the above descriptions, the significances of the decision-making method proposed in this paper are outlined as follows:

(1) From a theoretical perspective, our decision-making method is a combination of the TOPSIS method, the WAA operator method and CFRS theory. Therefore, our decision-making method has the advantages of these three theories in dealing with MCDM problems. At the same time, our ranking decision-making method can be efficiently applied to MCDM problems in a finite FCAS.

(2) From an application perspective, in Section 5.2, we provide a rating scheme for some merchants or companies. Our ranking decision-making method is the core of this scheme and can provide optimal sorting scheme information for the rating work. This approach is also beneficial to some merchants or companies to launch some new businesses in the future, such as selecting VIP customers.

7. Experimental analysis

There are two purposes for any sorting decision-making methods, namely the best alternative and the optimal sorting scheme. Based on these two purposes, this section is divided into two parts to conduct data experiments on our method. Through these two experiments, we can demonstrate the superiority and randomness of performance of our method. These two experiments are enforced by using MATLAB R2014a and accomplished on a personal computer with an Intel Core i5-4590, 3.30 GHz CPU, 4.0 GB of memory, and 32-bit Windows 7.

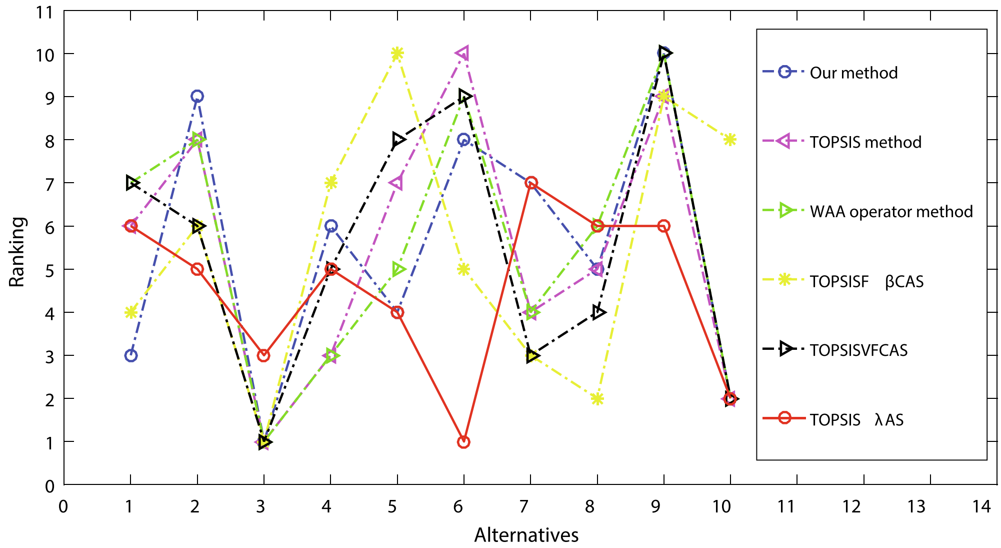

Fig. 3. The comparison of sorting results under different methods.

Table 18
The ranking results of all alternatives on different methods.

Different methods	Ranking of all alternatives
The WAA operator method [28]	$k_3 \succ k_{10} \succ k_4 \succ k_7 \succ k_5 \succ k_8 \succ k_1 \succ k_2 \succ k_6 \succ k_9$
The TOPSIS method [12]	$k_3 \succ k_{10} \succ k_4 \succ k_7 \succ k_8 \succ k_1 \succ k_5 \succ k_2 \succ k_9 \succ k_6$
The TOPSISFβCAS [38]	$k_3 \succ k_8 \succ k_7 \succ k_1 \succ k_6 \succ k_2 \succ k_4 \succ k_{10} \succ k_9 \succ k_5$
The TOPSISVFCAS [13]	$k_3 \succ k_{10} \succ k_7 \succ k_8 \succ k_4 \succ k_2 \succ k_1 \succ k_5 \succ k_6 \succ k_9$
The TOPSISλAS [33]	$k_6 \succ k_{10} \succ k_3 \succ k_5 \succ k_2 \approx k_4 \succ k_1 \approx k_8 \approx k_9 \succ k_7$
Our method	$k_3 \succ k_{10} \succ k_1 \succ k_5 \succ k_8 \succ k_4 \succ k_7 \succ k_6 \succ k_2 \succ k_9$

7.1. Experimental analysis with respect to the optimal sorting scheme

In Section 5.3, we use our ranking method to calculate the multi-criteria data set (Table 6) and obtain the final sorting. Note that this multi-criteria data set is obtained based on a collection of alternatives $M_1 = \{k_1, k_2, \cdots, k_{10}\}$. In this section, we will expand the capacity of the multi-criteria data set (i.e., increase the number of alternatives) to verify the superiority of our method's performance. According to the sample extraction method and data conversion method in Section 5.3, we obtain the following table.

Let $M_1 = \{k_1, \cdots, k_{10}\}$, $M_2 = \{k_1, \cdots, k_{20}\}$, $M_3 = \{k_1, \cdots, k_{30}\}$, $M_4 = \{k_1, \cdots, k_{40}\}$, $M_5 = \{k_1, \cdots, k_{50}\}$ represent different collections of alternatives, respectively. That is to say, M_1, M_2, M_3, M_4, M_5 represent 5 data sets. Using our method, we can calculate the sorting results of these 5 data sets as shown in Table 20.

Table 20 shows the ranking schemes L_1, L_2, L_3, L_4, L_5 of M_1, M_2, M_3, M_4, M_5, respectively. In order to illustrate the invariance of the original ordered relations in different collections of alternatives, Fig. 4 shows the comparison among L_1, L_2, L_3, L_4 and L_5.

From Fig. 4, we see that when the capacity of the data set increases, the original ordered relations still exist. For example, the ordered relation $k_3 \succ k_{10}$ exists on L_1, L_2, L_3, L_4 and L_5. Through the above analysis, we find that the performance of our ranking method is excellent. Moreover, our ranking method can also be applied to big data problems.

We continue to examine the randomness of performance of our method.

7.2. Experimental analysis with respect to the best alternative

Based on Table 19, we randomly sample 20 collections of alternatives $Q_1, Q_2, \cdots, Q_{20}$. Let the data sets corresponding to the collections of alternatives $Q_1, Q_2, \cdots, Q_{20}$ be $E_1, E_2, \cdots, E_{20}$, respectively. Besides, for every collection of alternatives, there are 5 alternatives which contain the alternative k_3. In Section 7.1, we find that when our method is applied to different data sets, the best alternative is consistent, i.e., k_3. In this section, we apply our method to these 20 data sets to acquire 20 optimal alternatives. If these 20 optimal alternatives are still k_3, then the randomness of performance of our method can be illustrated. The detailed ranking results of 20 collections of alternatives are shown in Table 21.

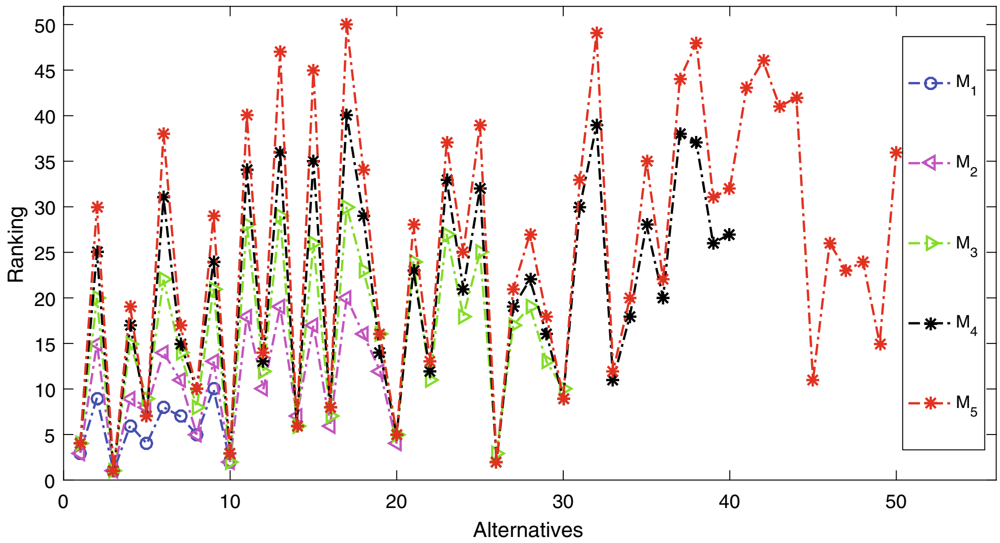

Fig. 4. The comparison of sorting results under different collections of alternatives.

Table 19
MCDM matrix with fuzzy information.

$W/\mathbf{H}$	H_1	H_2	H_3	H_4	H_5	H_6
k_1	1	0.6400	0.2500	0.9300	0.4200	0.5900
k_2	0.5600	0.7600	0.2900	0.9700	1	0.6500
k_3	0.8800	1	0.7100	0.7800	0.5900	0.7600
k_4	0.6600	0.8300	1	0.6700	0.4700	0.8200
k_5	1	0.4800	0.8200	0.5400	0.6900	0.8400
k_6	0.3900	0.2600	0.9800	1	0.9900	0.9600
k_7	0.4600	0.9500	0.7300	1	0.8500	0.5300
k_8	0.9800	0.6200	1	0.3700	0.6400	0.5100
k_9	1	0.4600	0.5800	0.1600	0.8800	0.5600
k_{10}	0.8500	0.5800	1	0.8800	0.9600	0.6100
k_{11}	0.8100	0.4700	0.3800	0.1700	1	0.6400
k_{12}	1	0.3800	0.8800	0.6100	0.3800	0.3700
k_{13}	0.6700	0.7500	0.5300	0.74600	0.2400	1
k_{14}	1	0.5200	0.5700	0.7700	0.4900	0.5800
k_{15}	0.2400	0.6900	1	0.8700	0.8500	0.3500
k_{16}	0.7500	1	0.2800	0.8600	0.1300	1
k_{17}	0.2700	0.4600	0.7700	0.8400	1	0.8700
k_{18}	0.5900	0.6400	0.7500	0.3900	1	0.5500
k_{19}	1	0.3400	0.3800	0.2500	0.5700	0.6200
k_{20}	1	0.4700	0.5800	0.8100	0.5800	0.5800
k_{21}	0.1500	1	0.0700	0.4300	0.2800	0.7700
k_{22}	1	0.6700	0.0500	0.0600	0.3500	0.3000
k_{23}	0.7200	0.5100	0.5800	0.1800	1	0.4700
k_{24}	0.4800	0.2600	1	0.3800	0.5400	0.2300
k_{25}	1	0.0100	0.3000	0.1400	0.4300	0.8400
k_{26}	0.1400	0.4400	0.1900	1	0.1600	0.1900
k_{27}	0.4800	1	0.5800	1	0.6100	0.2900
k_{28}	1	0.3400	0.4400	0.5700	0.7700	0.1700
k_{29}	0.7900	0.5300	1	0.5900	0.6400	0.2200
k_{30}	1	0.2800	0.3300	0.1400	0.4900	0.4300
k_{31}	0.5700	0.7500	0.1200	1	0.5400	0.3100
k_{32}	0.5700	0.5100	0.7900	0.2100	0.2900	1
k_{33}	0.9100	0.5000	1	0.7700	0.4300	0.5800
k_{34}	1	0.9100	0.5200	0.5200	0.1900	0.4800
k_{35}	0.6700	0.8900	0.5600	0.4800	0.6800	1
k_{36}	0.7700	1	0.6000	0.0700	0.3500	0.9700
k_{37}	0.7100	0.5200	0.2600	0.2300	1	0.4900
k_{38}	0.2200	0.8600	1	0.3300	0.5600	1
k_{39}	0.5500	1	0.6200	0.3900	0.7200	0.2100

Table 19 (*continued*)

$W/\mathbf{H}$	H_1	H_2	H_3	H_4	H_5	H_6
k_{40}	0.1700	0.2500	1	0.2400	0.8100	0.4700
k_{41}	0.6000	0.8400	0.4500	0.4100	1	0.4900
k_{42}	0.1800	0.4300	1	0.4700	0.7700	0.2600
k_{43}	0.6900	0.8100	0.2200	1	0.6800	0.2800
k_{44}	0.4600	0.2400	0.33	1	0.4300	0.7100
k_{45}	0.7100	1	0.2400	0.4900	0.4400	0.2200
k_{46}	0.2300	0.3500	0.8200	0.9300	1	0.1400
k_{47}	0.6800	0.1600	1	0.3700	0.5500	0.2700
k_{48}	0.3100	0.2100	1	0.9000	0.5100	0.3100
k_{49}	1	0.6100	0.8200	0.3900	0.8100	0.4200
k_{50}	0.4400	0.4700	1	0.1200	0.7800	0.5900

Table 20
The ranking results of alternatives on different collections of alternatives.

Different collections	Ranking of alternatives
$M_1 = \{k_1, k_2, \cdots, k_{10}\}$	$k_3 \succ k_{10} \succ k_1 \succ k_5 \succ k_8 \succ k_4 \succ k_7 \succ k_6 \succ k_2 \succ k_9$
$M_2 = \{k_1, k_2, \cdots, k_{20}\}$	$k_3 \succ k_{10} \succ k_1 \succ k_{20} \succ k_8 \succ k_{16} \succ k_{14} \succ k_5 \succ k_4 \succ k_{12} \succ k_7 \succ k_{19} \succ k_9 \succ k_6 \succ k_2 \succ k_{18} \succ k_{15} \succ k_{11} \succ k_{13} \succ k_{17}$
$M_3 = \{k_1, k_2, \cdots, k_{30}\}$	$k_3 \succ k_{10} \succ k_{26} \succ k_1 \succ k_{20} \succ k_{14} \succ k_{16} \succ k_8 \succ k_5 \succ k_{30} \succ k_{22} \succ k_{12} \succ k_{29} \succ k_7 \succ k_4 \succ k_{19} \succ k_{27} \succ k_{24} \succ k_{28} \succ k_2 \succ k_9 \succ k_6 \succ k_{18} \succ k_{21} \succ k_{25} \succ k_{15} \succ k_{23} \succ k_{11} \succ k_{13} \succ k_{17}$
$M_4 = \{k_1, k_2, \cdots, k_{40}\}$	$k_3 \succ k_{26} \succ k_{10} \succ k_1 \succ k_{20} \succ k_{14} \succ k_5 \succ k_{16} \succ k_{30} \succ k_8 \succ k_{33} \succ k_{22} \succ k_{12} \succ k_{19} \succ k_7 \succ k_{29} \succ k_4 \succ k_{34} \succ k_{27} \succ k_{36} \succ k_{24} \succ k_{28} \succ k_{21} \succ k_9 \succ k_2 \succ k_{39} \succ k_{40} \succ k_{35} \succ k_{18} \succ k_{31} \succ k_6 \succ k_{25} \succ k_{23} \succ k_{11} \succ k_{15} \succ k_{13} \succ k_{38} \succ k_{37} \succ k_{32} \succ k_{17} \succ$
$M_5 = \{k_1, k_2, \cdots, k_{50}\}$	$k_3 \succ k_{26} \succ k_{10} \succ k_1 \succ k_{20} \succ k_{14} \succ k_5 \succ k_{16} \succ k_{30} \succ k_8 \succ k_{45} \succ k_{33} \succ k_{22} \succ k_{12} \succ k_{49} \succ k_{19} \succ k_7 \succ k_{29} \succ k_4 \succ k_{34} \succ k_{27} \succ k_{36} \succ k_{47} \succ k_{48} \succ k_{24} \succ k_{46} \succ k_{28} \succ k_{21} \succ k_9 \succ k_2 \succ k_{39} \succ k_{40} \succ k_{31} \succ k_{18} \succ k_{35} \succ k_{50} \succ k_{23} \succ k_6 \succ k_{25} \succ k_{11} \succ k_{43} \succ k_{44} \succ k_{41} \succ k_{37} \succ k_{15} \succ k_{42} \succ k_{13} \succ k_{38} \succ k_{32} \succ k_{17}$

Table 21
The ranking results of alternatives on different collections of alternatives.

Different collections of alternatives	Ranking of alternatives
$Q_1 = \{k_1, k_3, k_5, k_7, k_9\}$	$k_3 \succ k_5 \succ k_7 \succ k_1 \succ k_9$
$Q_2 = \{k_1, k_2, k_3, k_4, k_5\}$	$k_3 \succ k_5 \succ k_4 \succ k_1 \succ k_2$
$Q_3 = \{k_3, k_4, k_5, k_6, k_7\}$	$k_3 \succ k_5 \succ k_4 \succ k_6 \succ k_7$
$Q_4 = \{k_3, k_8, k_9, k_{10}, k_{11}\}$	$k_3 \succ k_8 \succ k_{10} \succ k_9 \succ k_{11}$
$Q_5 = \{k_3, k_{12}, k_{13}, k_{14}, k_{15}\}$	$k_3 \succ k_{14} \succ k_{12} \succ k_{15} \succ k_{13}$
$Q_6 = \{k_3, k_{16}, k_{17}, k_{18}, k_{19}\}$	$k_3 \succ k_{16} \succ k_{19} \succ k_{17} \succ k_{18}$
$Q_7 = \{k_3, k_{20}, k_{21}, k_{22}, k_{23}\}$	$k_3 \succ k_{20} \succ k_{21} \succ k_{22} \succ k_{23}$
$Q_8 = \{k_3, k_{24}, k_{25}, k_{26}, k_{27}\}$	$k_3 \succ k_{27} \succ k_{25} \succ k_{26} \succ k_{24}$
$Q_9 = \{k_3, k_{28}, k_{29}, k_{30}, k_{31}\}$	$k_3 \succ k_{28} \succ k_{30} \succ k_{29} \succ k_{31}$
$Q_{10} = \{k_3, k_{32}, k_{33}, k_{34}, k_{35}\}$	$k_3 \succ k_{34} \succ k_{33} \succ k_{35} \succ k_{32}$
$Q_{11} = \{k_3, k_{36}, k_{37}, k_{38}, k_{39}\}$	$k_3 \succ k_{36} \succ k_{39} \succ k_{38} \succ k_{37}$
$Q_{12} = \{k_3, k_{40}, k_{41}, k_{42}, k_{43}\}$	$k_3 \succ k_{43} \succ k_{41} \succ k_{42} \succ k_{40}$
$Q_{13} = \{k_3, k_{44}, k_{45}, k_{46}, k_{47}\}$	$k_3 \succ k_{45} \succ k_{44} \succ k_{47} \succ k_{46}$
$Q_{14} = \{k_3, k_{47}, k_{48}, k_{49}, k_{50}\}$	$k_3 \succ k_{49} \succ k_{50} \succ k_{47} \succ k_{48}$
$Q_{15} = \{k_3, k_{13}, k_{23}, k_{33}, k_{43}\}$	$k_3 \succ k_{33} \succ k_{43} \succ k_{13} \succ k_{23}$
$Q_{16} = \{k_3, k_{11}, k_{21}, k_{31}, k_{41}\}$	$k_3 \succ k_{21} \succ k_{41} \succ k_{31} \succ k_{11}$
$Q_{17} = \{k_3, k_{12}, k_{22}, k_{32}, k_{42}\}$	$k_3 \succ k_{12} \succ k_{22} \succ k_{32} \succ k_{42}$
$Q_{18} = \{k_3, k_{14}, k_{24}, k_{34}, k_{44}\}$	$k_3 \succ k_{14} \succ k_{34} \succ k_{44} \succ k_{24}$
$Q_{19} = \{k_3, k_{15}, k_{25}, k_{35}, k_{45}\}$	$k_3 \succ k_{45} \succ k_{35} \succ k_{15} \succ k_{25}$
$Q_{20} = \{k_3, k_{16}, k_{26}, k_{36}, k_{46}\}$	$k_3 \succ k_{16} \succ k_{36} \succ k_{26} \succ k_{46}$

Table 21 illustrates that our method is applied to these 20 collections of alternatives whose optimal alternatives are consistent with the known optimal alternative k_3. This phenomenon is a powerful demonstration of the randomness of our method's performance.

8. Conclusion

In this paper, we firstly define two pairs of CFRS models and study their properties and relationships. Second, we propose a new MCDM method in a finite FCAS. Furthermore, we provide a rating scheme based on an MCDM method for some merchants or companies. Then, to illustrate the rationality and superiority of this new method, we use two classic methods and three latest methods to compare with our method. Finally, we also verify the performance of our method. The core points of this paper are as follows:

(1) We construct models $\left(\underline{F}_{N^{\mathbf{H}}_{\lambda},\mathscr{T},\mathscr{I}},\overline{F}_{N^{\mathbf{H}}_{\lambda},\mathscr{I},\mathscr{T}}\right)$ and models $\left(\underline{F}_{N^{\mathbf{H}}_{\lambda},\mathscr{I},\mathscr{I}},\overline{F}_{N^{\mathbf{H}}_{\lambda},\mathscr{T},\mathscr{T}}\right)$. Then, based on different fuzzy neighborhood operators, we study the nature and relationship of the models.
(2) We study a new MCDM method in a finite FCAS, which is a combination of the TOPSIS method, the WAA operator method, and CFRS theory. At the same time, in order to classify some customers more effectively, we design a rating scheme. The core of this rating scheme is an MCDM method in a finite FCAS.
(3) We use five decision-making methods to compare with our methods and illustrate the rationality and superiority of our method. The reason is that all five methods can realize the decision in a finite FCAS. Through method comparison, we can show the effectiveness and feasibility of our method. In addition, from the perspectives of the best alternative and optimal ranking scheme, we verify the performance of our method.

In the future, we want to consider the following ideas for research. Firstly, we hope to explore other properties and applications of models $\left(\underline{F}_{N^{\mathbf{H}}_{\lambda},\mathscr{T},\mathscr{I}},\overline{F}_{N^{\mathbf{H}}_{\lambda},\mathscr{I},\mathscr{T}}\right)$ and models $\left(\underline{F}_{N^{\mathbf{H}}_{\lambda},\mathscr{I},\mathscr{I}},\overline{F}_{N^{\mathbf{H}}_{\lambda},\mathscr{T},\mathscr{T}}\right)$, such as attribute selections [4], attribute reductions [11], feature selections [30], granular computing [23], lattices [41], and so on. Secondly, we hope to develop more efficient MCDM methods in a finite FCAS.

Declaration of Competing Interest

The authors declared that they have no conflicts of interest to this work.

CRediT authorship contribution statement

Kai Zhang: Conceptualization, Methodology, Investigation, Writing - original draft. **Jianming Zhan:** Conceptualization, Methodology, Investigation, Writing - original draft. **Xizhao Wang:** Writing - review & editing.

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